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Devoted to the interests of mathematics in Elementary and Secondary Schools

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THE MATHEMATICS TEACHER

Volume XXXV Number 1

Edited by William David Reeve

An Addition to the Mathematics Attainment Test of the College Entrance Examination Board

In June 1942 and thereafter the Mathematics Attainment Test held by the College Entrance Examination Board will consist of the following four sections:

- Mathematics 2A. Two-Year Mathematics—principally algebra (Alternative Alpha)
- Mathematics 2B. Two-Year Mathematics—principally algebra and geometry (Alpha)
- Mathematics 3. Three-Year Mathematics (Beta)
- Mathematics 4. Four-Year Mathematics (Gamma)

Mathematics 2B, 3, and 4 have previously been known as Mathematics Alpha, Beta, and Gamma, respectively. Mathematics 2A is a new examination, alternative to Mathematics 2B. It was prepared by the Board's Committee of Examiners in Mathematics in response to a demand from a number of schools that the Board provide an examination suitable for candidates whose first two years of mathematics have been devoted primarily to the study algebra.

The following description of Mathematics 2A, together with the introductory paragraph distinguishing it from Mathematics 2B, will be included in the December 1941 edition of the *Description of Examination Subjects:**

Mathematics 2A and 2B

Mathematics 2A and 2B are designed for candidates who have had one year of elementary algebra and one additional year of mathematics. Mathematics 2A is designed for those candidates whose additional year of mathematics has been concerned principally with second year algebra. Mathematics 2B is designed for those candidates whose additional year of mathematics has been concerned principally with demonstrative geometry.

Mathematics 2A (Alternative Alpha)

The examination will be chiefly concerned with the concepts and techniques of arithmetic and algebra. It will include some questions also on easy intuitive geometry, numerical trigonometry of the right triangle, and logarithms, but it will be so constructed that a candidate cannot obtain a higher score by sacrificing thorough competence in the fundamentals of arithmetic and algebra in order to show

* Upon request a single copy will be sent to any teacher without charge. To others the charge for a copy will be thirty cents, which may be remitted in postage stamps. Address requests to the College Entrance Examination Board, 431 West 117th Street, New York, N. Y.

acquaintance with a wider range of topics. Further indication of the scope of this examination is furnished by the following list of topics, but this list is not offered as a complete syllabus for a course of instruction.

1. Understanding of important ways of employing convenient symbolism for the communication of ideas. Skill in arithmetical operations and algebraic manipulation.
   (a) Number systems (e.g., natural numbers, signed numbers, rational numbers, real numbers).
   (b) Symbolism and vocabulary (e.g., coefficients, exponents, use of parentheses, use of subscripts).
   (c) Laws of operation (e.g., rational operations in various number systems).
   (d) Techniques (e.g., ratio and proportion, the summing of progressions, the use of the binomial theorem).
   (e) Ability to generalize from particular instances and to apply general rules to particular cases.
2. Understanding of functional dependence, its mathematical formulation and its applications.
   (a) The formula (e.g., formulas which appear in geometry, in physics, or in other fields within the experience of the candidate).
   (b) The equation (e.g., linear and quadratic equations in one variable, systems of equations in several variables, distinction between an equation and an identity).
   (c) The graph (e.g., representation of statistical data and of relations between two variables given by equations).
3. Problem solving, i.e., the translation of a question into algebraic form, its solution and interpretation.
4. Intuitive geometry.
   (a) Simplest notions concerning triangles, quadrilaterals, regular polygons, perpendiculars, parallels, and the circle. No formal proofs will be required.
   (b) Knowledge and understanding of the facts of mensuration.
5. The solution of right triangles by the use of four-place tables (natural functions and logarithms). Some understanding of the technique of calculation, of the importance of checks, and of limitations imposed upon the accuracy of results by the use of approximate data and of tables.

Geometry

By ALICE MONTGOMERY, *High School, Collinsville, Tenn.*

It is, according to the text, a study of form,
Of efficiency of methods
Used in dealing with objects,
Their position, their shape, their size.
Why does the text stop there?
It is so cold that way,
Only a branch of mathematics,
If the text only enlarged to tell
All that geometry underlies!

Oh, it should mention stars and suns and snow-
flakes,
And tell of cobwebs,
They give meaning
To what is alone, a senseless word.

It should use hearts of rocks and jewels and
rain
Of flowers or ferns
Make reality of what was definition
Uninteresting, monotonous, and blurred.

A study of form—yes, of arcs of stars,
Of atoms,
Of the infinitesimally small and great,
What makes for the eternal stillness
Or neverceasing strife.
Geometry is the study of form
But as the study of form
Geometry is the study of the Universe,
Of Life.

Models in Solid Geometry

By Miles C. Hartley
University High School, Urbana, Illinois

The development of the pupil's ability to visualize spatial relationships has for a long time been recognized as one of the problems confronting the teacher of Solid Geometry. In 1923, the National Committee on Mathematics Requirements wrote: "The aim of the work in Solid Geometry should be to exercise further the spatial imagination of the student and to give him both a knowledge of the fundamental relationships and the power to work with them."[1] In 1940, the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics reported: "Much attention should be given to the visualization of spatial figures and relations, to the representation of three dimensional figures on paper . . . ."[2]

The practice of making models with cardboard, wire, and string to illustrate the early theorems in the unit on *Lines and Planes* is quite common. The assumption has then been made that thereafter no assistance in the visualization of three-dimensional figures is needed, but experience has shown that many pupils have difficulty throughout the course because they are unable to "see" the figures. This situation has led the Mathematics Department at the University High School to introduce into the Solid Geometry course a unit on *Models*. A description of the unit follows.

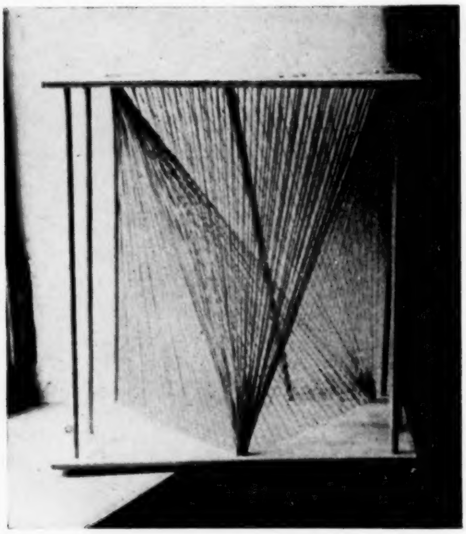
The materials for the models consisted of ply-wood, dowels, carpet warp, glue, and nails. Since ply-wood warps easily, care was taken to see that it was as true as possible; it was one-quarter of an inch thick and was cut into foot squares. For Cavalieri's Theorem two pieces, two feet by one foot, were used. The dowels were three-eighths of an inch in diameter and were cut into pieces twelve inches long. Balls of carpet warp in several colors were purchased. Small three-quarter inch nails were used.

Since the unit on *Models* was introduced at the conclusion of the unit on *Lines and Planes*, it was necessary to define first polyhedron, prism, parallelopiped, pyramid, cylinder, and cone, for each of which a small wooden model was available and was presented to the pupils to aid them in understanding the definitions. Next, a list of theorems was given to the class and each individual member chose a theorem for which he wished to make a model. The list included such theorems as:

1. If two solids have equal altitudes, and if sections formed by planes parallel to the bases and at equal distances from the bases have equal areas, the solids are equal in volume.
2. The volume of a triangular pyramid is equal to one-third the product of its base and altitude.
3. An oblique prism is equal to a right prism whose altitude is equal to a lateral edge of the oblique prism and whose base is a right section of the oblique prism.
4. The lateral area of a prism is equal to the product of a lateral edge and the perimeter of a right section.
5. If a pyramid is cut by a plane parallel to the base, the section is similar to the base.
6. A plane through two diagonally opposite edges of a parallelopiped divides the parallelopiped into equal triangular prisms.
7. Every section of a cylinder made by a plane passing through an element is a parallelogram.
8. In a circular cone a section made by a plane parallel to the base is a circle.
9. The intersection of a right circular cone and a plane is a conic section.

[1] *The Reorganization of Mathematics in Secondary Education,* A Report by the National Committee on Mathematical Requirements under the Auspices of The Mathematical Association of America, Inc., 1923, p. 36.

[2] *The Place of Mathematics in Secondary Education,* Fifteenth Yearbook of the National Council of Teachers of Mathematics, New York: Bureau of Publications, Teachers College, Columbia University, 1940, p. 97.



Theorem 2A—Volume of a Triangular Pyramid

The next step might be called exploration; here each pupil familiarized himself with his theorem, being sure that he understood the meaning of the theorem and that he knew the definition of all terms involved. He made at least three drawings of the figure as seen from different angles and also a drawing of the model (without strings.) Finally, he made a plan for the model. For Theorem 6, above, one position of the figure (Fig. 1), the drawing of the model (Fig. 2), and the plan (Fig. 3) would appear as at the bottom of this page.

When these preliminaries had been completed, the entire class went to the woodshop where they finished their projects. First, they executed their plans (Fig. 3) in pencil on the squares of ply-wood. After doing this, they drove nails at about half-inch intervals along the pencil lines and pulled them out. Next, they glued and nailed four dowels to the bottom piece at the points marked A and then they attached the top in the same manner. Finally, they laced the carpet warp through the holes, using one color of warp for the lateral surface and a different color for the intersecting plane.

This unit required one week and then the work proceeded through the customary units on prisms, cylinders, pyramids, and cones. When the class encountered a theorem for which a model had been made, the pupil who had constructed the model was chairman of the class for that particular day. He was expected to have complete mastery of the theorem, i.e., to be able to answer any question proposed by members of the class and to be able to give a correct demonstration of the theorem. This procedure stimulated the poorer members of the class who did their best work when using their models. The reactions of the entire class were very

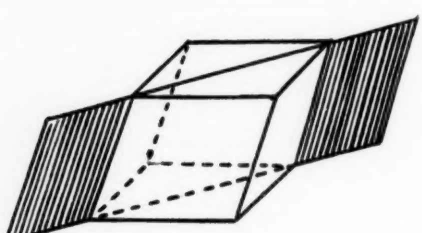


Fig. 1

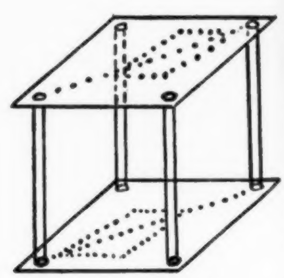


Fig. 2

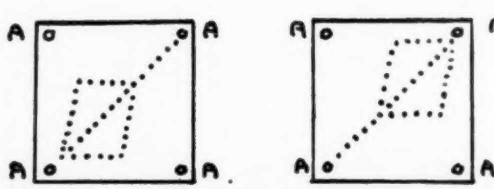
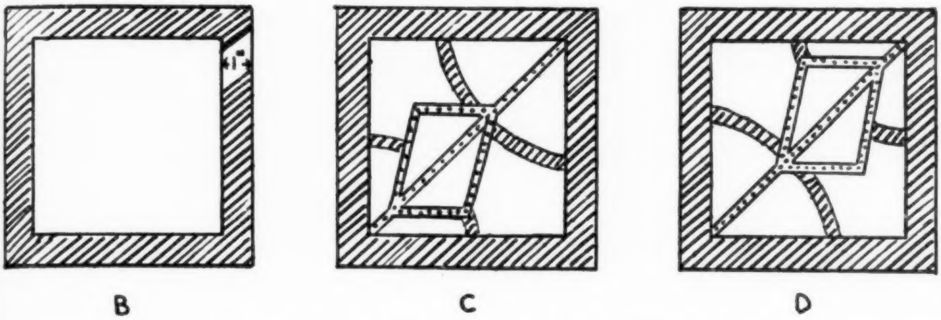


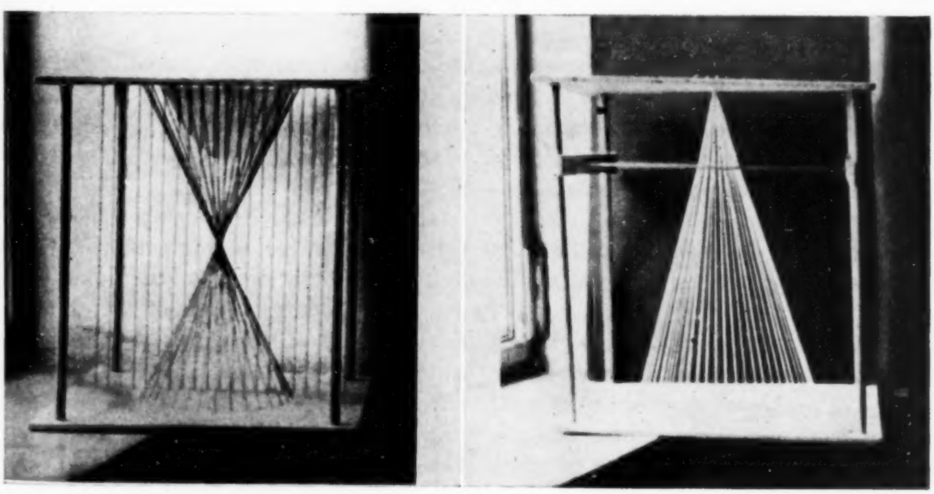
Fig. 3

favorable, and the results were most satisfactory. At no time during the semester did we hear the familiar phrase, "I can't see that."



Plans for Frame work of Brass Model of Theorem 6

The procedure indicated here is not the only possible one. After the preliminary instructions have been given in class, the models can be completed as an out-of-



Theorem 9—Conic Sections

The Hyperbola The Circle

school project, or one class period a week (or part of a period) can be devoted to the construction of models.

The expense involved is not excessive since the average cost per model was only eleven cents. If better and more permanent models are desired, they can be constructed of brass and strung with silk thread.

Models of brass will, of course, require some knowledge of, and skill in, sheet metal work. For these models, sheets of brass, twelve inches wide and one-twelfth of an inch thick, can be purchased. Six pieces twelve inches square are required for each model. In the case of the model of the intersection of a parallelopiped and a plane (described above), there would be four pieces like *B* and one each of *C* and *D*. Holes for the string are drilled easily, and the six pieces are then soldered together to form the box-like framework of the model, which can be painted if desired.

A Mathematics Diagnostic Testing Program in Purdue University, II

The Error Analysis of the Number Technique Test

By M. W. Keller, D. R. Shreve, *and* H. H. Remmers
Purdue University

The first paper[1] on the diagnostic testing program in mathematics listed the problems used on the test, the student ability rating, and the student accuracy rating on each pair of matched problems. In addition a few comments on students' difficulties were given. It was hoped that this information would be a definite help to teachers of secondary mathematics by indicating those specific topics on which the students showed lack of ability or understanding when they entered the university, so that those topics might receive more emphasis. It was also hoped that university instructors would consider such factual findings in planning college courses in elementary mathematics with a view to including, either in a preliminary review or in the course proper, those topics which are particularly necessary to satisfactory achievement in freshman college mathematics and which are usually assumed by the instructor to be known, yet which are not understood by a large part of the students.

The purpose of this paper is to give in detailed tabular form an error analysis for each type of problem included in the test, with the frequency of occurrence of each type of error, and to discuss the types of errors characteristic of each type of problem. It is felt that this information will indicate many, perhaps most, of the more common types of errors which students make on problems of the types included in the test. Thus it is likely that a number of difficulties and incorrect concepts, especially those which are most prevalent, can be eliminated (or minimized in frequency) as those topics are taught for the first time, either in primary or secondary education. This information might also be used profitably as a basis for remedial teaching.

The original error analysis was made in considerable detail. In some cases the errors of a particular type occurred only occasionally; since the frequency of occurrence was not sufficient to be significant, those errors were grouped under a 'Miscellaneous' heading. (In this group were 16 cases in fraction problems where the denominator had been omitted from the answer, and 19 errors which were made in reducing to a common denominator.) The rest of the errors were found to be conveniently classified in 11 groups. This classification is given in the 'Error Analysis Code.'

In the 'Error Frequency Table' the problems have been grouped with respect to type of operation involved so that it can be seen immediately what types of errors are characteristic, let us say, of multiplication, and which are characteristic only of certain kinds of numbers involved in the multiplication process. In order that the table shall be as informative as possible, one problem of each of the matched pairs is given, and the same problem number is assigned to each pair as in the previous paper. The matched problems of a pair involved the same fundamental manipulations and in the judgment of the authors were of equal difficulty. That they were of equal difficulty (except for one division problem pair) was verified from the results obtained on the test by comparing the numbers of students who ob-

[1] M. W. Keller, D. R. Shreve, H. H. Remmers, A Report on a Diagnostic Testing Program at Purdue University. The Mathematics Teacher, Nov., 1940, pp. 321–324.

NUMBER TECHNIQUE ERROR ANALYSIS CHART

(279 Students)

No.	Problem Type	Order	Error Code No. 0	1	2	3	4	5	6	7	8	9	10	11	Total Errors
1.	$63 \times \frac{8}{9}$	5, 38	2, 4	2	8	6			2	2	1	1		11	33
9.	$41 \times \frac{5}{7}$	9, 46	5, 11	6		13		1	7	4		1	27	26	85
14.	$97.6 \times .0708$	1, 34	0, 4	2	42	26	25	14	5	23					137
10.	$\frac{15}{91} \times 0$	13, 50	3, 13	49											49
32.	$7.9 \div 0$	3, 48	0, 9	416											416
15.	$\frac{0}{43.7}$	7, 52	6, 15	55											55
17.	$\frac{7}{18} \div \frac{13}{18}$	10, 29	5, 10	20	6	1			14	9		26	1	2	79
7.	$\frac{3}{7} \div \frac{5}{8}$	6, 25	3, 4	26	9	2			25	1	1		9	3	76
20.	$\frac{44}{35} \div \frac{33}{40}$	18, 37	11, 15	28	8	21			5	4		57		39	162
25.	$\frac{19}{11} \big/ \frac{19}{8}$	14, 33	18, 27	53	5	1				4		52	1		116
8.	$\frac{35.7}{.07}$	30, 53	3, 15					48	2	3				15	68
28.	$0.78 \div 0.0089$ (to 1 dec.)	26, 49	9, 25	26	35	8	10	35	4	6			136	8	268
31.	$3.798 \div 8.7$ (to 3 dec.)	22, 45	2, 17	5	36	21	22	26	5	5			283		403
23.	$0.84 \div \frac{3}{7}$	36, 61	14, 26	18	9	8		37	47	1		14	5	16	155
13.	$\frac{5}{39} + \frac{2}{13}$	15, 64	2, 23	10		9	7		10	2	3	16		1	58
18.	$\frac{13}{23} - \frac{17}{23}$	11, 60	10, 25	17			31		10	9	5				72
3.	$\frac{4}{15} - \frac{2}{9}$	4, 19	7, 4	12			2		7	4	26	13			64
2.	$(3)^2$	17, 62	0, 16	18						1					19
12.	$(.2)^2$	2, 21	0, 0	22				52							74
21.	$\left(\frac{\sqrt{5}}{3}\right)^2$	32, 63	19, 28	41					28						69
6.	$\sqrt{\frac{16}{81}}$	20, 35	16, 13	30		16				1					47
19.	$\sqrt{\frac{5}{9}}$	28, 51	31, 37	56							8	15	4		83
29.	Simplify: $\frac{3}{\sqrt{2}}$	24, 47	33, 41	118					16					5	139
4.	17% of 93	8, 23	3, 6	11	13	8	11	8	23	6					80
24.	7% of $\frac{7}{3}$	16, 31	15, 27	37		3		24	31	1			38	18	152
22.	What per cent of 50 is 9?	12, 27	7, 18	107				1		1				24	133
16.	Write .007 as a fraction	39, 54	18, 36	29											29
27.	Write .007 as a per cent	40, 55	23, 35	116											116
5.	Write $\frac{3}{8}$ as a decimal	41, 56	11, 32	6				7							13
11.	Write $\frac{3}{8}$ as a per cent	42, 57	13, 32	2				22							24
30.	Write 2.7% as a fraction	43, 58	36, 47	107								23			130
26.	Write 2.7% as a decimal	44, 59	44, 53	6				77							83

tained the correct answer to each of the paired problems. The coefficient of correlation on the entire test of 'odds' vs. 'evens' was 0.936; the Spearman-Brown prophecy formula gave 0.967 as the predicted coefficient of correlation for two full-period tests.

In the Error Analysis Chart the first column gives the number of the problem pair, the numbering being the same as in the first paper. In the second column is given one problem of the matched pair. In the third column the positions of the pair of problems on the test are given. In the next column, marked 0, the number of students who omitted each problem of the pair is given. Thus in the first row, the figure 1 refers to problem pair number 1;

the problem is $63 \times 8/9$; the first problem of the pair appeared as number 5 on the test, the second problem of the pair appearing as number 38 on the test; 2 students omitted problem 5 on the test, and 4 omitted problem 38. Although it is not possible to say definitely whether a problem was omitted because of lack of time or because of lack of understanding, in some cases it is evident that the problem was omitted because of lack of understanding, since only 23 omitted the last problem (number 64) on the test. The numbers in the fifth to fifteenth columns indicate the frequency of occurrence of the errors of the types indicated at the tops of the columns. The entries in the 'Total Errors' column indicate the total number of errors made on the two problems of the matched pair. So we find in the first row that there were 33 errors on the two problems of pair number 1 on the papers of 279 students, or 33 errors on 558 problems.

From the error chart it is seen that in problem pairs 1, 9, and 14 almost all the students understood the correct procedure. The mistakes were due mostly to inability to perform accurately the operations indicated. In problem type 14, 10% of the errors were due to a misplaced decimal point, and 4% to a misinterpretation of the symbol of operation. Errors of these types, however, cannot be classified as purely mechanical errors. The decimal point errors certainly should be considered as a kind of "lack-of-knowledge" error, while in most instances, since the symbols of indicated operations are certainly well-known, the "misinterpretation of symbol" errors were probably for the most part due to carelessness. In problem pair 9 another kind of error is seen,—the inability to "round off" an answer correctly. In this pair of problems about 32% of the errors were of that nature. These errors cannot be classified as carelessness or "due to a lack of knowledge of the correct procedure" in the indicated operation, but to a lack of knowledge of what is meant by writing an answer correct to any specified number of places or significant figures. It seems that the student believes that if he has the number 12.374, it is quite legitimate and proper, if the answer is to be written only to one decimal, to write 12.3 as the answer, ignoring the last two digits. Whether this is actually the case or whether the student just doesn't think that the number might be better approximated by 12.4 when performing the division is open to question. This point will be investigated in subsequent tests.

In problem pair 10 in every case except one, when the answer "infinity" was given, the student gave the fraction 15/91 as the answer. This indicates that there are quite a few students who can perform a multiplication by zero, as in problem pair 14, in a purely mechanical way without understanding the fundamentals of multiplication by zero.

Problem pair 32 displays in a very striking fashion the fact that the fundamental concept that division by zero is an excluded operation has certainly not been sufficiently emphasized. There were 310 responses of 0 and 106 responses of the dividend 7.9 as the answer. In addition, there were less than 1% of the students who gave the really correct answer (that the division is impossible); the remainder of the answers which were counted correct were either the word "infinity" or the symbol ∞.

In the next pair, number 15, again it is seen that the incorrect answer indicates a definite lack of knowledge of what is meant by division, as there were 14 answers of "infinity," 26 answers of the denominator 43.7, and the remaining answers constituted a miscellaneous assortment of incorrect concepts. In each case it was evident that the difficulty was caused partially by a hazy notion of the fundamental definition of division; that is, if a is the quotient of $c \div d$, then $ad = c$.

Of the students who apparently did not understand the correct procedure in pairs 17 and 7, 13 and 15 respectively inverted the dividend instead of the divisor, while

3 and 5 respectively inverted both dividend and divisor. A rather disturbing fact was that even when there was an "obvious" common factor which should be "cancelled," on 26 occasions the students failed to perform so simple a reduction in performing the division. In pair 17, the students added on 12 occasions and multiplied the other 2 times. On the other pair, the students multiplied 6 times, added 15 times, and subtracted 4 times. This should not be due to misreading symbols, since the problems which were division problems were specifically pointed out to the students after they had received their papers and just prior to starting the test. (The students were asked to mark carefully these problems *as division problems* so they would not be read as problems in addition.)

In pair 20 we see that approximately the same errors persist. Thus there are still about the same number who do not proceed correctly. Since the numbers are larger (and consequently there were more mechanical operations to perform correctly before obtaining the right answer), it is to be expected that there would be more of these types of errors. The results were not disappointing. What was surprising to the authors in view of the two preceding problem pairs was that there were only 5 misinterpretations of the symbol of operation. There is apparently no way to account for the difference.

Pair 25 indicates that a division problem written as a complex fraction with the vinculum written obliquely presents difficulties from the view of understanding the meaning of the manipulations to be performed. Thus, more perform incorrect operations on this pair than other division problems, and more omit them than other problems in the vicinity of these problems. Among the students who know how to operate, there is still that relatively large group who do not divide out the most obvious common factors. About 50% of the errors on these problems were failures to simplify.

There is some inability to locate a decimal point correctly in a division problem, as is indicated in type 8. The other errors are probably, for the most part, due either to carelessness or attempting to perform the division mentally when it was too much for students to do correctly in that manner.

In problem pairs 28 and 31, there is a larger group of manipulative errors. This indicates very concretely a weakness of the students: their inability to perform a slightly involved numerical computation correctly. There is perhaps one solution to this difficulty, and that is plenty of drill emphasizing accuracy, whether with or without reduction of speed. The errors in Column 1 were all due to reversing the dividend and divisor. Again the students show a complete lack of knowledge of how to "round off" correctly. This is shown in a very convincing fashion where, in pair 28, one problem could be written by carrying the division to one decimal and disregarding the remainder. In the other problem of the pair, the remainder was more than one-half after the quotient had been found to one decimal place. On this problem there were 129 errors in approximating the quotient.

The last of the division problems (type 23) involved a division of a decimal by a fraction. There was a scattering of errors with a rather large number of "incorrectly placing the decimal point" and "misinterpretation of symbol" errors.

In the addition and subtraction problems there were relatively few errors. It is, perhaps, a trifle discouraging to realize that after students had been "exposed" to all of grade school arithmetic and to a minimum of two semesters of algebra there were still 39 instances in which the student did not know how to proceed correctly in problems as simple as these, and also that there were students who omitted these simple problems. Of the 31 subtraction errors, 22 were failures to indicate that the result was negative.

In problem types 2 and 12, there were a

total of 40 times that the students failed to understand the meaning of an exponent. In 19 instances the student considered the exponent as a multiplier; that is, 2^3 was equivalent to 2×3, and the other 21 times the students indicated the work for 2^3 as $2\times2=4$, and $4\times4=16$, and gave 16 as the answer. Here again we find a large number of decimal point errors even when the multiplication was correct otherwise.

Problem types 21, 6, and 19 revealed all sorts of false concepts of how to proceed. There were no common types of misunderstanding. Each student seemed to have a unique misunderstanding of the correct procedure. In the 28 errors of misinterpretation of the symbol in pair 21 the students considered the fraction to be under the radical.

In pair 29 the students were given an opportunity to decide what was meant by simplifying the expression. Some rationalized the denominator only; others rationalized the denominator, then extracted the square root and divided; others extracted the square root and performed the division. A correct answer obtained in any of these manners was counted correct if the work was done without any manipulative error. Once again there was a great diversity of incorrect procedure. The incorrect answer which was given most frequently was $\frac{9}{2}$. This answer was given 40 times. From the students' papers it was evident that those who gave this particular incorrect response thought that if a number was squared, the square was still equal to the original number. In 18 instances they gave the problem as the answer and in 10 instances they gave $\frac{3}{2}$. No work was on the paper for the problem when the latter answer was given; so it was not possible to determine how the students obtained that answer.

In the first of the percentage problem pairs, approximately one-half of the errors were mechanical. The 8 decimal point errors were the result of the students not understanding the meaning of per cent. Of the 23 errors in failing to recognize that "of" indicated multiplication, all the students divided. Most of the students divided the number by the per cent. Ten of the errors classed as lack of knowledge gave the correct numerical values, but added a per cent sign to the answer, as 15.81%.

The second pair, number 24, of the per cent problems involved the same operation as the preceding pair, except that one of the numbers was a fraction. This doubled the number of errors, though there were definitely fewer mechanical errors. The 37 errors classified as "lack of knowledge" involved such confusion and inconsistency that it was impossible to classify them in any satisfactory manner. The decimal point errors were again due to a lack of understanding; thus the decimal point and misinterpretation of symbol errors were of the same kind as in the preceding pair of problems. One should also notice that the inability to "round off" correctly persists whenever a student finds such an error possible.

The last pair, number 22, of this group indicates very clearly that the concept of percentage is quite vague to a large number of students. Of the 107 errors in column 1, there were 28 who took 9/100 of 50 to get the answer, there were 50 who divided by 9, there were 19 who gave the answer .18 or 9/50, and there were 10 who seemed to guess at the answer.

In problem type 16 there are few errors. Most of these apparently were caused by the student reading the decimal incorrectly, and giving the answer 7/100. In converting the same decimal to per cent, there were 4 times as many errors as occurred when the decimal was to be converted to a fraction. There were 47 who gave the answer .007%, 36 who gave the answer .07%, 15 the answer 7%, 6 the answer $\frac{1}{7}$, and the remainder gave a miscellaneous assortment of incorrect responses.

Problem pair 5 had very few errors while, when such a well known fraction is to be converted not to a decimal but to

per cent, we see a marked increase in the number of errors. Almost all the errors were due once again to a lack of a clear understanding of per cent. Most of the incorrect responses gave the number .75%.

The last two problem types present no new types of errors. The types of errors are the same as in the other problems of this group, with the exception of the 23 cases where the student did not simplify. In this case the students left the answer as 2.7/100. This answer, though not incorrect, could certainly not be considered as in simplified form or as completely converted to a common fraction. When asked to write 2.7% as a decimal, the majority of the students who gave the answer as 2.7 did not know how to place the decimal point; and when asked to write 2.7% as a fraction, 64 gave the answer as $2\,\frac{7}{10}$, and 23 gave 27/100.

The general error trends in certain types of problems suggest that if the teachers will stress these points and maintain and use properly an adequate diagnostic testing program throughout the work, an improvement may be effected in the pupil's grasp of the fundamentals. There should be particular emphasis on understanding not only *how* to do a certain process, but also *why* it is done.

There is one obvious conclusion that may be drawn,—that even though the fundamentals are well understood, if there is no adequate maintenance program, not only through a given year but throughout the students' entire academic career, forgetting has already left large gaps in the students' understanding and ability by the time they reach the university. Apparently, then, some sort of a maintenance program should be carried on. Though not sufficient, perhaps, but a distinct advance, would be a review of all the fundamentals of arithmetic during the senior year in high school, especially for those planning to continue their training in universities and trade schools.

Two attempts to prepare entering freshmen for success in university work by a preliminary review, as suggested in the first paragraph of this paper, were begun by Purdue University during the summer of 1940. In collaboration with some of the larger city school systems of Indiana, intensive review courses in pre-college mathematics were offered in strategically located centers, to provide an opportunity for interested high school teachers to resolve some of their students' difficulties, which could not be done in regular secondary course work. To meet the needs of those students who did not have access to the established centers of review instruction, a correspondence course was offered by the University. A full report on the findings and achievements of this latter course will be given in later papers.

ERROR ANALYSIS CODE

Code *Explanation*

0 The problem was omitted. No attempt was made to determine those who omitted because of lack of knowledge of the correct procedure and those who omitted the problem because of lack of time.

1 Did not understand the correct procedure. This means that an incorrect answer was given, and the method was also incorrect.

2 Multiplication error in "carrying over." For example: $76\times8=588$. The student either added incorrectly or lost a figure.

3 Multiplication error. For example: $708\times9=6381$. That is, the student multiplied 8×9 as if it were 9×9.

4 Addition or subtraction error.

5 Decimal point incorrectly placed.

6 Misinterpreted symbol of operation. That is, the students added when division was indicated, divided when multiplication was indicated, etc.

7 Careless errors. Wrote a 7 or 2 that looked like a 1, then used it as a 1, etc., or miscopied.

8 Miscellaneous errors. Various errors

which occurred too infrequently to be significant enough to classify separately.

9 Did not simplify. This occurred in fractions where the common factor was not divided out.

10 "Rounded off" incorrectly. For example, in the problem $0.68 \div .0079$ the answer was required to 1 decimal. In the division, the quotient was 86.075. So the answer should have been 86.1 while the student usually just divided until the first decimal, zero, was obtained and hence the answer was written 86.0.

11 Division or cancellation error.

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What Does It Cost to Own and Operate an Automobile?

By Gaylord C. Montgomery
John Burroughs School, Clayton, Missouri

The automobile appears to be an indispensable part of the present mode of civilization. Its use and misuse are presented in newspapers, magazines and school textbooks. America is truly motorized. From conversations with my acquaintances I learn that frequently the difference between having a financial reserve from the annual income and having an unbalanced budget is due to the decisions made relative to the cost of ownership and operation of an automobile.

Have your students, after solving a typical textbook problem, expressed the opinion that their solution was wrong because the answer seemed unreasonable? For example, the speed of a plane, from data included in the problem, may be only 90 miles per hour, or the average speed of an automobile may be only 23 miles per hour. After such an experience with a textbook problem which was concerned with the cost of driving an automobile on a vacation trip, my students asked, "What does it cost to own and operate an automobile?"

Estimates made by the ninth-grade students of the cost per mile to own and operate an automobile ranged from $0.025 to $0.25. When their estimates were recorded on the blackboard, a demand came to know which one or what is the correct answer.

Desiring to enlist their efforts in a study of the problem, I declined to offer my estimate but asked for a list of the factors relevant to the solution. The costs of gasoline, oil, and tires appeared on the list of each student, of course. On several lists, repairs, insurance, and taxes were included; depreciation, washing, and simonizing on others. Several students expressed the belief that an accurate and complete record was kept at home and that an accounting of the receipts would provide a relatively simple solution. Permission was secured by two students to summarize and submit to the class the cost items for which bills had been paid. Our list of factors involved became longer, but two factors which I desired to include had not been suggested and time was taken for a discussion of them. These were the cost of home garaging and the lost or "imputed" interest on the investment. This latter item seldom appears in a textbook but its understanding has been listed as an objective of mathematics teaching.[1]

If a car is kept in a garage at home, the man who has his own garage must buy or build it, keep it in repair, and pay taxes on it. Rent for a house with a garage on the lot is greater than one with no garage, in my experience. The students agreed that the cost of the garage should be counted as a cost of owning and operating the car. If a man pays $500 or $700 or $1200 for an automobile, he should realize that he might, instead of buying the car, have used the money to buy a bond, stock, or otherwise invest it so that a return on his investment would accrue at a rate of, say, 4%, which would amount to $20 or $28 or $48, respectively. The students agreed that the "imputed" interest should be included as a cost of owning and operating the automobile.

Some discussion was also necessary with regard to the depreciation factor, the amount and kinds of insurance carried, and city and state licenses. The costs were classified as variable and fixed (constant). Sometimes a family wonders whether to use the car for a vacation trip, or to leave

[1] Christofferson, H. C., et al., "Ohio Curriculum Guide," Bulletin No. 52, 1934. State Department of Education, Columbus, Ohio.

the car at home and go on the train or bus. In figuring the cost of using the car in this case, only the variable costs should be counted. The fixed costs go on whether the car is used or left at home, and they should not be included in counting the extra cost of using the car on the trip.

Finally, our list of relevant factors involved numbered eighteen, and the next step was to assign or secure from reliable records the charge for each item. Is the number of years that a car is kept by one owner a factor in its cost and operation? Does the number of miles that the car is driven affect the trade-in value of the car? Does the depreciation involved spread the first cost of the car over its term of ownership? If so, is it the same each year? Is the anti-freeze jugged in the spring and used the following winter? These questions illustrate a few of the minor problems encountered by the students in securing data pertinent to the solution of the major problem.

Following is a summary submitted by a student:

Cost Items	*First Year*	*Second Year*	*Third Year*
Gasoline	$222	$255	$256
Oil	16	21	26
Lubrication	13	13	13
Insurance	63	63	63
Depreciation	339	226	190
Garage	36	36	36
Imputed Interest	46	32	23
Seat Covers	16		
Tires		38	40
Repairs		40	70
Licenses	16	16	16
Property tax	22	17	14
Parking	47	50	50
AAA membership	12	12	12
Tolls	8	6	8
Anti-freeze	4.50	4.50	4.50
Simonizing	10	10	10
Washing	18	15	12
Totals	$888.50	$854.50	$843.50

"Data in the above table are with regard to a Buick purchased in 1938 for $1140. Gasoline cost is computed on the average of 11 miles per gallon, using a regular grade of gasoline purchased at Standard Oil Company stations. Our tires are replaced at 16,000 miles. Oil is changed at 1000 mile intervals and none added between changes during the first year. Two motor tune-ups annually are made. The annual mileage is about 13,000 miles. Anti-freeze is drained into the sewer in the spring. A vacation trip during the second year in states where the price of gasoline was higher than at home accounts for the increased cost for that item. Dividing the annual cost by the number of miles driven during the year, I find that the cost per mile is about $0.06 for the first year and about $0.065 for each of the second and third years."

Work toward the solution depends upon the analysis of the data collected.[2] Some of the data was discarded because it was irrelevant; data retained needed organization to study it effectively.[3] The students agreed that a tabular arrangement was advisable. Incidental to experiencing and understanding the nature of the problem-solving process, various fundamental elements of the process were encountered. For example, on about the fourth day after the problem was formulated, I listed on the board a fragment of the data collected to date and requested each student to list the possible conclusions that seemed reasonable from an examination of the data presented. The next day I listed another fragment, and, later in the week, still another. Practice was thus afforded in attempting analyses of increasing difficulty, as the data successively presented required more careful examination. A sample follows:

Make of Car	*Year Model*	*Yearly Cost per Mile* *1*	*2*	*3*	*4*
Chevrolet	1939	.06	.06	.04	.04
Chevrolet	1939	.06	.05	.05	.04
Chevrolet	1938	.05	.05	.04	.04
Chevrolet	1936	.15	.12	.10	.09

Conclusions listed after an examination of the data stimulated a discussion of the variable and fixed costs. It was then that we learned that a summary made by one student included an item of $1080 as the salary paid to the chauffeur who drove but one of their automobiles. This one factor more than doubled the cost per mile compared to the cost per mile if that item was omitted.

[2] *The Place of Mathematics in Secondary Education* (Bureau of Publications, Teachers College, Columbia University, New York) 1940, p. 23.

[3] *Mathematics in General Education* (New York, D. Appleton-Century Co., 1940, p. 92.

Each student selected the make of automobile—Ford, La Salle, Hudson, Studebaker, Lincoln-Zephyr, Nash, Chevrolet, Mercury, etc., owned at home, and from actual records available, supplemented by agreements reached from our discussions and authoritative printed materials, an individual compilation of costs was made. A summary similar to that previously given was prepared by each student, and surprise was expressed when an examination of summaries posted on the bulletin board revealed that graphical methods had been used to organize data. Those students using only tabular forms immediately wanted to known why others had included bar and circular graphs, for example.

There is evidence to support the opinion that content mastered in the solution of worth-while problems is mastered and retained more permanently than content mastered for its own sake.[4] The work evolving from the solution of this problem served as motivation for all work done with statistical graphs. "Interest in a subject can result from study of it as well as motivate it."[5]

The results obtained in the study of this problem were checked by a committee of students with the records kept at the local AAA office. Information supplied by families from which the students came revealed a surprising acquaintance with publications dealing with problems of the consumer. Many of the students will soon be driving an automobile, and our activities as described supplied them with new and useful information.

Mathematics is one of several fields which have an opportunity to develop habits of intelligent thinking by students. To formulate and solve problems intelligently, students must be confronted with whole problems.

[4] *Science in General Education* (New York, D. Appleton-Century Co.), 1938.

[5] *The Place of Mathematics in Secondary Education* (Bureau of Publications, Teachers College, Columbia University, New York) 1940, p. 27.

On the Use of Models

I MENTION one other method—that of the use of models. In many schools an extensive use of manufactured models is constantly made; in more pasteboard models of home manufacture are provided; in some the opposite extreme is followed, and not models only but blackboard diagrams also are banished from the recitation, the class forming mental images of the figure and projecting them perhaps on the floor, and possibly better yet, from the point of view of united attention, on the teacher. My own idea is that both these extremes have in them somewhat of good, and, if too persistently used, much evil. Certainly, if geometry is to find its way to the lower grades, the use of models is helpful and essential, though most of them may well or better be of home manufacture, worked out together with the solution of the principle. In the high school they have their place, and under present conditions no collection is likely to be too fine or too expensive.—From an article "On Modern Methods in Geometry" in *School Review,* 1896, pp. 70–79.

Teachers interested in work like the above should send any suggestions they may have to Dr. E. H. C. Hildebrandt, State Teachers College, Upper Montclair, N. J., who is chairman of the committee which is preparing the 18th yearbook of the National Council of Teachers of Mathematics relating to multi-sensory aids in teaching mathematics.—Editor.

Scope of Mathematical Offerings in Selected Junior Colleges*

By Lorella Ahern
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Perhaps no one of those subjects which serve both the preparatory and terminal functions has as great a range of possible channels by which to serve these functions as has mathematics. Even a cursory enumeration of the avenues by which mathematics may fulfill many terminal and preparatory functions causes one to be confronted by two questions. Is the junior college including the numerous opportunities for valuable preparatory and terminal accomplishments that are embodied in the mathematical fields? If not, are many of these opportunities being neglected because of a lethargic attitude displayed by mathematics departments which proffer a potentially extensive subject matter in a narrow traditional manner?

Despite the great need for investigation of the mathematics that is being offered by the junior college little investigation has been attempted.

The purpose of the investigation reported here is to examine the status of mathematics in the junior college as reflected by the official catalogues of selected public junior colleges located in the area in which the North Central Association of Colleges and Secondary Schools functions.

In order to facilitate an analysis of the findings and to permit comparison of data pertaining to colleges of comparable size, the colleges studied have been separated into four classifications. These classifications are based upon the enrollment of the particular colleges as listed in the Directory of American Junior colleges for 1938–1939. All colleges which have a total enrollment of less than 99 are classified in Group A, those having an enrollment which falls between 100 and 349 inclusive are classified in Group B, enrollments falling between 350 and 599 inclusive are classified in Group C, and enrollments of 600 or more are classified in Group D.

Table I shows the location and the number of colleges included in this study.

Even a perfunctory examination of junior colleges gives evidence of the extreme variation in the position which mathematics occupies in the program of studies.

The survey of mathematics in the junior college which is set forth in this report is concerned only with the scope of the courses offered. The statements and the data submitted are based upon titular description exclusively. Only in instances where ambiguity or erroneous inference might otherwise result is there any attempt to base interpretation upon course content.

The data yielded by the investigation of the course content, irrespective of the agreement or disagreement of titulary indication, constitute an amount of material which cannot be included in this brief article.

Apropos the extent of the mathematical offerings in the junior colleges with which this investigation is concerned, Table II reveals that 529 courses are listed by the 82 institutions under 59 course titles. This table shows, too, how frequently each course is offered by the total number of colleges, and by the colleges in each group.

The data in Table II would seem to suggest the following interpretations: The tendency for College Algebra and Trigonometry to be offered with greater persistency than other subjects is quite consistent throughout the different groups of colleges. There is, however, one subject that is repeatedly encountered almost to the

* Summary of a chapter from a Master's Thesis submitted in the Department of Secondary Education at the University of Nebraska.

TABLE I

Colleges Classified as to State and Grouping Employed in This Study; Also the Per Cent of Total Colleges Represented in Each State

State	Total Public Colleges in State	Number of Colleges Studied	Number of Colleges in Group A	B	C	D	Per Cent of Public Colleges Studied in Each State
Arizona	2	2		1		1	100%
Arkansas	7	4		2	1	1	57
Colorado	4	2		2			50
Illinois	8	8		3	1	4	100
Indiana	1	1		1			100
Iowa	27	16	11	5			59
Kansas	14	9		5	4		64
Michigan	9	5		3	1	1	55
Minnesota	12	9	1	5	3		75
Missouri	10	7	2	4		1	70
Nebraska	2	2	2				100
New Mexico	2	1			1		50
North Dakota	3	3	1	1		1	100
Oklahoma	29	11	3	6	1	1	38
South Dakota	1	1		1			100
West Virginia	1	1		1			100
Total	132	82	20	40	12	10	62

same extent that are College Algebra and Trigonometry. This is Analytic Geometry. It is offered by 60 of the 82 colleges studied, while College Algebra and Trigonometry are each offered by 63 of the colleges. Only these three subjects occur with sufficient frequency to justify inclusion in any preconceived idea of course offerings which might reasonably be anticipated in the mathematics department of a public junior college.

Another course in mathematics which occurs frequently, in one or the other of two forms, is Calculus. It is most often presented in courses labeled Differential or Integral Calculus. Though this differentiation, in title at least, is common, there are many colleges which offer what may be designated as a general course in Calculus. This consists of a course, of one or of two semesters, described as Calculus, unencumbered by further titular limitations or descriptions. In all but five of the 25 instances in which it was offered, there was also offered a continuation course of this subject which was similarly titled. In these courses various topics from either branch of the Calculus are introduced indiscriminately. Differential Calculus is offered by 47 colleges, and all but three of these offered Integral Calculus. Combining these data yields the fact that 72 colleges offer one semester of work in the Calculus, and 64 offer two semesters of work in the same subject.

It must not be inferred from the foregoing statement that Calculus is offered more often than are College Algebra, Trigonometry, or Analytic Geometry. There are a number of titles, such as Introduction to College Mathematics, General Mathematics, etc., which embrace the usual work in Algrbra, Trigonometry, and Analytic Geometry. There are, too, courses such as College Algebra-and-Trigonometry which occupy the time of one semester and which are followed by such a course as Trigonometry-and-Analytic Geometry. Quite obviously these courses cover the same area of work but it is equally obvious that they cannot be added to the number of courses in any one of these subjects without introducing fallacious totals, yet neither can they be ignored. For these reasons they are considered separately and are mentioned here only to prevent any suggestion that these three subjects, College Algebra, Trigonometry, and Analytic Geometry, are offered by only some 60 colleges.

TABLE II

Frequency of Course Offerings in Mathematics Departments of 82 Public Junior Colleges

Title of Course	Group A No.	Group A %	Group B No.	Group B %	Group C No.	Group C %	Group D No.	Group D %	Total No.	Total %
College Algebra	12	60	34	85	11	92	6	60	63	77
Trigonometry	12	60	33	83	11	92	7	70	63	77
Analytic Geometry	9	45	34	85	11	92	6	60	60	73
Differential Calculus	9	45	24	60	8	67	6	60	47	57
Integral Calculus	8	40	22	55	8	67	6	60	44	53
Higher Algebra	6	30	16	40	5	42	4	40	31	38
General Calculus	5	25	12	30	4	33	4	40	25	30
(Cont'd) General Calculus	5	25	8	20	3	25	4	40	20	24
Solid Geometry	2	10	10	25	3	25	1	10	16	20
Algebra and Trigonometry	11	55	4	10			1	10	16	20
Trigonometry and Analytic Geometry	7	35	2	5					9	11
Business Mathematics	3	15	1	2.5	1	8	4	40	9	11
Mathematics of Investment			5	12.5	3	25			8	10
Plane and Solid Analytic Geometry	1	5	3	7.5	1	8	1	10	6	7
Descriptive Geometry	1	5	2	5	1	8	2	20	6	7
Introduction to College Mathematics			2	5	1	8	3	30	6	7
General Mathematics	1	5	4	10	1	8			6	7
Technical Mechanics			2	5	3	25			5	7
Algebra and Analytic Geometry	2	10	1	2.5	1	8	1	10	5	7
Mathematics of Finance					3	25	2	20	5	7
Elementary Statistics			3	7.5	1	8	1	10	5	7
Advanced Arithmetic	1	5	2	5	1	8			4	5
Survey Course in Mathematics			1	2.5			3	30	4	5
Surveying	1	5	3	7.5					4	5
Spherical Trigonometry	1	5	2	5			1	10	4	5
College Algebra for Engineers			3	7.5	1	8			4	5
Plane Geometry			1	2.5	2	17			3	4
Trigonometry for Engineers			2	5	1	8			3	4
Engineering Problems	1	5	1	2.5	1	8			3	4
Analytic Mechanics			1	2.5	1	8	1	10	3	4
(Cont'd) Introd. to College Mathematics							3	30	3	4
Commerce Algebra			1	2.5	2	17			3	4
College Arithmetic			1	2.5			1	10	2	2
Calculations			1	2.5			1	10	2	2
Statics and Kinematics			2	5					2	2
School Algebra and Trigonometry			1	2.5			1	10	2	2
Solid Analytic Geometry			2	5					2	2
Analytic Geometry for Engineers			1	2.5	1	8			2	2
Practical Mathematics	1	5	1	2.5					2	2
Slide Rule			1	2.5	1	8			2	2
(Cont'd) Engineering Problems	1	5							1	1
(Cont'd) Analytic Geometry			1	2.5					1	1
College Algebra for Agriculture Students			1	2.5					1	1
Integral Calculus and Differential Equations			1	2.5					1	1
Accounting I and II			1	2.5					1	1
Algebra for Accountants			1	2.5					1	1
Theory of Equations and Solid Anal. Geom.			1	2.5					1	1
College Geometry					1	8			1	1
Differential Equations					1	8			1	1
Theory of Equations					1	8			1	1
Solid Mensuration					1	8			1	1
Algebra, Trigonometry and Anal. Geom.					1	8			1	1
Business Computation					1	8			1	1
School Algebra and Trigonometry for Medical, Dental and Forestry Students							1	10	1	1
(Cont'd) Above Course							1	10	1	1
Advanced Trigonometry							1	10	1	1
Unified Mathematics							1	10	1	1
Mathematics Preparatory to Statistics and Finance							1	10	1	1
Trigonometry for Electrical Students							1	10	1	1
Total Number of Courses	100		255		97		76		529	
Total Number of Titles	22		44		34		30		59	
Number of Schools in Group	20		40		12		10		82	

READ TABLE THUS: College Algebra is offered in 12, or 60%, of the schools in Group A; in 34, or 85%, of the schools in Group B; etc.

Still another course which displays such a marked tendency to a general scattering of recurrence as to merit attention at this point is Higher Algebra. It is offered by 31 schools. This frequency alone constitutes one claim to consideration, for it is sufficiently great to give Higher Algebra sixth place among the subjects listed according to frequency. It is also one of the 13 subjects which are offered by all four groups of colleges. In the great majority of cases this subject is offered under the title Higher Algebra; however, in the colleges in Group B, 16 of which offered this subject, in five instances the title was Intermediate Algebra, and in two instances the subject was referred to as School Algebra. The title School Algebra occurs in a few instances in connection with combination courses.

There is a definite tendency for the junior colleges to present mathematics courses, the presentation of which has been considered the prerogative of the high school.

Less than one-fourth of the subjects found in the mathematics departments are offered by the colleges in all groups. These 13 subjects which are offered by at least one college in all groups are confined to the basic and traditional mathematics courses of college and high school, with the exception of three courses, namely, Descriptive Geometry, Business Mathematics, and the combination course of Algebra and Analytics. Ten mathematics courses are offered to aid the student who is going into the field of commerce, but Business Mathematics is the only course offered by the colleges in all groups, and is also the course which is most frequently offered. Accounting is listed in Table II and would bring the total to eleven, if it were counted. Accounting had to be listed, for this study is concerned with the subjects listed in the mathematics departments of the junior colleges in question; however, at the same time it is recognized that Accounting is listed in the commercial departments in the other institutions.

Comparatively popular in the junior college are the mathematics courses which may be described as survey, or general courses. This type of course is offered under one of five titles. There are also eight titles which suggest that the material which is usually treated in a specific field of mathematics has been combined with that from another field.

TABLE III

Range of the Number of Courses Offered in the Mathematics Departments of 82 Junior Colleges

	Groups				Total
	A	B	C	D	
Median number of mathematics courses	4.78	6.3	7.5	7.0	6.18
Range in number of courses in mathematics	2–10	2–12	4–13	5–13	2–13
Quartile deviation	.75	1.5	1.5	1.12	1.6

A consideration of Table II, from the viewpoint of the grouping used throughout this study, shows that the colleges in Group A, which are the smallest colleges, offer 22 separate courses, those in Group B, 45 courses. The number of colleges in Group A is only half the number in Group B, but it would hardly seem that this fact accounts for the great variation in the number of courses offered, for the range in the number of courses offered is very similar for these two groups. The range for all groups is shown on Table III. Groups C and D differ little in the number of courses offered or in the range in number of courses offered.

The number of hours credit offered by these colleges ranged from 5 to 42. The lowest median number of hours is offered by the Group A colleges, and the highest median number by the colleges in Group

C. The median number of hours credit offered by the entire number of colleges is slightly over 24.

Summary

There is wide variation in the number of mathematics courses offered by the junior colleges, which has little relation to the saze of the collge in question.

The median number of mathematics course offered is greatest in the colleges which are classified in Group C. Fifty-nine course titles were found in the mathematics departments of the colleges studied. The number of titles was greatest in the colleges classified in Group B.

There is little similarity displayed by the pattern of courses offered in the mathematics departments of the junior colleges studied, beyond some form of College Algebra, Trigonometry, Analytic Geometry and the Calculus.

Mathematics courses are offered for students who are preparing for specific professions and for specific vocations.

In many instances a course is offered by one college, but is not offered by any of the other colleges included in the study.

The Use of Arithmetic in the Out-of-School Life of Children

By Roy DeVerl Willey
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The utility arithmetic research studies of Wise, Woody, Thorndike, Mitchell, Charters, Schorling, Bobbitt, Wilson and Bowden were all concerned with the use of arithmetic by adults.[1] These studies have done much in eliminating superfluous arithmetic from our public schools, but they have also resulted in a questionable practice of imposing adult standards on pupils with little consideration of immediate needs and interests. Valuable as the results of the studies of adult usage have been, they cannot and should not continue to dominate our curricula without proper supplementation by research in child usage.

Research in child usage has thus far been scanty, and the results which have been found were derived only by indirect methods which dealt largely with the very young child. The studies which have been made of child arithmetic usage may be divided into those dealing with use out of school and those dealing with use in school. This article will deal with studies of out-of-school use only.

The number of concepts of very young children as studied by child psychologist are perhaps the first studies which were made which are indirectly related to out-of school usage. These studies deal principally with the abilities children possess in number concept. To include them as studies related to child-use we must make the assumption that knowledge of how numbers are used means that use of these numbers has some time or another been made.

One of the earliest psychological studies made of child-use of number was that of Alfred Binet[2] in 1890. His experimentation with small numbers caused him to conclude that the perception of number for the four-year-old was "four" and, after some practice, "five." A two-year-old child could not recognize with assurance numbers beyond three. He concluded that the child has very great difficulty in perceiving a mass in any other than continuous form, even when he should perceive it as a large number of units.

In 1897 Phillips[3] wrote an excellent account on numerical concept, where he gave the results of research on the knowledge of young children of number. Phillips experimented on kindergarten children and consulted primary teachers. He collected data from 616 persons by means of a questionnaire. He concluded that children first name the series without reference to objects of any kind. Every attempt to instruct children in numbers shows that the series-idea is highly abstract. On making a test of thirty-nine children in the kindergarten, he found that thirty-three of them counted without the slightest reference to the objects to be counted, always running the series far ahead of the objects.

Deucher reported his results of analyzing number concepts of children in the thirteenth volume of the Zeitschrift, 1912.[4] He analyzed these number concepts into five processes, and upon this analysis he arranged a series of tests for children from three to seven years of age.

[1] For summaries of these various studies, see Wilson, Guy M., *What Arithmetic Shall We Teach?* Houghton Mifflin Company, Boston, 1926.

[2] Binet, Alfred. "La perception des longeurs et des nombres," *Revue Philosophique,* Vol. 30 (Juillet 1890), pp. 68–81.

[3] Phillips, D. E. "Number and Its Application Psychologically Considered," *Pedagogical Seminary,* Vol. 5 (Oct. 1897), pp. 221–281.

[4] Deucher, G. "Psychologische Vorfragen des ersten Rechnen Unterrichts," *Zeitschrift. f. Pädagogische Psychologie,* Vol. 13 (1912), pp. 36–52.

About this same time (1912) Decroly[5] gave a most detailed and consecutive report of the different evidences of understanding number relations which his daughter showed from her fourteenth to her fifty-eighth month. Decroly devised a series of tests based upon his observations of the way in which number ideas are manifested by young children from two and a half to six years of age. Descoeudres[6] made extensive use of these tests. She found that at two and a half years, children can repeat the numbers in order from 1 to 3 and that the proficiency in this regard increases regularly to the ability to count to 6 at four, to 8 at four and a half, and to 9 at five. Children of four years can count pebbles from 1 to 6 and those of five years to 10.

In 1919, Court[7] reported a study of number, time, and space in the first five years of one child's life. Typical abilities were: at two years of age the child named objects by pairs and made comparison of lengths; at three years the child could count to 12, count objects within 6, and knew the meanings of halves, quarters, pair, day, hour, minute, seasons, big, high, thick, fat, and lean; at four years the child became conscious of patterns and design and the general meaning of maps; and at five years the child could count to 1000, read numbers to 100, and add 6 plus 6. Fours years later Court[8] reported number concept of this child from the age of five to the age of eight. At the age of five this child could count to a million. The author reports that, had the child been left to himself and only encouraged and helped in his own arithmetical undertakings, he would have known nothing of the four fundamental operations at the age of eight. The age of five to eight is characterized by much less arithmetical spontaneous interest than from two to five. School may have satisfied this interest. Bad weather, which kept the child indoors, caused more spontaneous interest.

Although the study of Court reports results of the observation of one child only, his observations are very valuable for investigators who wish to study the out-of-school use of arithmetic by children more extensively.

Beckmann[9] (1923) studied the familiarity with numbers from one to five which was possessed by 465 children whose ages ranged from two to six years. All testing was done by objects such as small cubes, dice, marbles, etc. He found that reproducing or imitating numbers developed first and that number naming developed most slowly. Among other abilities quoted, children whose ages were two and a half had an average record for reproducing of 1.4 and for naming of .1, while the averages of the six-year-old were 4.66 and 4.07. Beckmann gave the components of number consciousness as (a) hearing numbers (b) speaking numbers (c) producing numbers (d) differentiating numbers (e) finding numbers and (f) naming numbers. The number activities of a child are his particular reactions to a definite situation, and cannot be comparable to the abstract number concepts of an adult. It may be possible for a child to reproduce a group of three objects without yet having mastered the ability to count three or having acquired the ability to name the group without counting.

Baldwin and Stecher[10] (1924) report re-

[5] Decroly, M. le Dr., and Degand, Mlle. Julia. "Observations relatives a l'evolution des notions de quantites, continues et discontinues chez l'enfant," *Archives de psychologie*, Vol. 12 (Mai 1912), pp. 81–121.

[6] Descoeudres, Alice. *Le Developpement de l'Enfant*, Paris, Delachaux and Niestle, S. A. (no date), pp. 233–280.

[7] Court, S. R. A. "Numbers, Time and Space in the First Five Years of the Child's Life," *Pedagogical Seminary*, Vol. 27 (1919), pp. 71–89.

[8] Court, S. R. A. "Self-Taught Arithmetic from the Age of Five to the Age of Eight," *Pedagogical Seminary*, Vol. 30 (March 1923), pp. 51–68.

[9] Beckmann, Hermann. "Die Entwicklung der Zahlleistung bei 2–6 jährigen Kindern," *Zeitschrift f Angewandte Psychologie*, Vol. 22 (1923), p. 20.

[10] Baldwin, Bird T., and Stecher, Loile. *The Psychology of the Pre-School Child*. New York, D. Appleton and Co., 1924, pp. 152–169.

sults of an experiment with children at the Child Welfare Station of the University of Iowa. In counting by rote, the average of the two-year-olds was 1, of the three-year-olds 3, of the four-year-olds 11, of the five-year-olds 24, and of the six-year-olds 25. These abilities were much higher than those earlier reported by Descoeudres.

In 1927 William Stern[11] published a review of other investigations along with his own. Part of his data appears in the following table:

AGES AT WHICH THE NUMBERS 2, 3, AND 4 ARE MASTERED

	2 years old	3 years old	4 years old
Beckmann	3.6 to 4	4 to 4.6	5 to 5.6
Fillbig	3.9	4.2	5.6
Descoeudres	3	4	5

Stern concludes that arithmetical development makes its greatest progress about the age of four, namely at the time when the child masters the distinction between 1 and 2. As soon as the 3 idea is conquered, the way is apparently clear for the higher numbers.

Buckingham[12] and MacLatchy[13] (1930) report a study of number ability of children when they enter grade one. Classroom teachers were asked to give tests to the first six of her pupils in the alphabetical list of surnames. The situation of a typical first grade class, the members of which have had previous kindergarten experience, are: (a) one child will not know how to count (b) one child will count to five (c) three children will count to ten (d) nine children will count to twenty (e) two children will count to thirty (f) three children will count to forty (g) two children will count to fifty (h) two children will count from fifty to ninety-nine and (i) three to one hundred.

A similar study reported by Woody[14] (1931) shows (a) that children have considerable knowledge of counting before formal instruction begins, (b) the exercises involving counting twenty circles and counting them in order prove easier than rote counting to 100 by 1's or 10's, (c) the exercise involving rote counting to 100 by 10's are easier than that involving counting to 100 by 1's. In general, the knowledge possessed by children is not limited to counting and adding simple combinations, but includes elementary knowledge of fractions, U. S. money, units of various types of measurement, and the understanding of the processes demanded in simple verbal problems.

In 1935 Polkinghorne[15] and McLaughlin[16] reported accounts of number ability of young children. Polkinghorne made a study of 266 children and found that they had certain concepts of fractions. These children gained much of this knowledge from their experiences at home and at school. The author advocates that primary children be exposed to the kinds of experience which bring about learnings in the group. The primary school curriculum in number should contain opportunities for children to acquire some of the simpler fundamental ideas about fractions in the form of concrete experiences with real objects.

McLaughlin[16] found that children of three years of age could count to five, while children of five years of age could count to thirty-three. It is difficult for children to combine objects into a group. Most of them count. Although it is more complex to enumerate objects in a series,

[11] Stern, William. *Psychologie der Frühen Kindheit*, Leipzig, Verlag von Qvelle and Meyer (1927), pp. 342, 416.

[12] Buckingham, B. R., and MacLatchy, Josephine. "The Number Abilities of Children When They Enter Grade One," *Twenty-Ninth Yearbook of the National Society for the Study of Education*. (Report of the Society's Committee on Arithmetic. Bloomington, Ill., Public School Pub. Co., 1930.) Pp. 473–524.

[13] MacLatchy, Josephine. "Number Abilities in the First Grade," *Childhood Education*, Vol. 11 (May 1935), pp. 344–347.

[14] Woody, Clifford. "The Arithmetical Background of Young Children," *Journal of Educational Research*, Vol. 24 (Oct. 1931), pp. 188–201.

[15] Polkinghorne, Ada R. "Young Children and Fractions," *Childhood Education*, Vol. 11 (May 1935), pp. 354–358.

[16] McLaughlin, Katherine. "Number Ability of Pre-School Children," *Childhood Education*, Vol. 11 (May 1935), pp. 348–353.

rote counting develops just slightly in advance of rational counting.

The discussion thus far has dealt with investigations dealing only indirectly with the child's out-of school use of number. The experiments reported deal almost entirely with the number ability of pupils as they have acquired this ability incidentally in their natural life situations. We now turn to the group of investigations which have thus far been made of the direct use children make of number in their out-of-school experience.

The first attempt at this kind of investigation is probably that of P. G. Noon[17] in 1919. His own statement is self-explanatory:

"Accordingly, I have gone to the children who know me and have asked them to tell me what uses of numbers they make or notice outside of school . . . . I went to twelve classes of grades four, five and six and to two immature classes of grade seven; in a most informal and friendly way I got them to give me the information I sought."

A tabulation of children's answers was made. The author credits each item to the lowest grade that reported it. A part of the items mentioned by pupils in grade four is given below:

(1) Games
  (a) calling and reading numbers on dominoes and dice
  (b) counting marbles and tops
  (c) keeping score
  (d) playing store
(2) Going to Store
  (a) quantity and price
  (b) counting change
(3) Other Instances
  (a) telephone and automobile numbers, street car numbers, house numbers, telling time and reading thermometer.

Noon concludes that children make little use of number outside of school and recommends that applied arithmetic in the grades below the Junior High School be discarded. Practical problems are as foreign as Chinese to a child's way of thinking.

The most frequently quoted study of this type is probably that of Smith[18] (1924). She attempted to discover by personal interview with 500 children in the Detroit schools the exact sort of arithmetic used by them outside of school. Thirty per cent of the occasions for the use of arithmetic occurred in transactions carried on in stores, eighteen per cent in games involving counting, fourteen per cent in reading Arabic numerals in finding pages in a book, six per cent in dividing food with playmates and pets, five per cent in depositing money in and drawing it out of toy banks and three per cent in playing store. These activities constituted eighty-nine per cent of the occasions for the use of arithmetic by pupils in the first grade. She also found that 35% of the operations were with addition, 23% counting, 12% subtraction, 9% fractions, 6% reading Arabic numerals, 5.5% measuring, 4.3% comparison, 3.1% reading Roman numerals, 2% multiplication and 1.1% division.

In 1938 Robinson[19] made a study to find what arithmetic problems are encountered by children in their home environment. Parents were asked to record their observations of problems used by their children in home life. The largest percentage of non-computational problems was found in grade one with the ratio of non-computational to computational problems becoming progressively less as grade level increases. Problems dealing with buying comprised one-fifth to one-third of the total situations in the various grades. Children's number needs relating to "time" were of considerable importance, especially in the lower grades. Measuring,

[17] Noon, P. G. "The Child's Use of Number," *Journal of Educational Psychology*, Vol. X, No. 9 (Nov. 1919), pp. 462–467.

[18] Smith, Nila B. "An Investigation of the Uses of Arithmetic in the Out-of-School Life of First-Grade Children," *Elementary School Journal*, Vol. 24 (April 1924), pp. 621–26.

[19] Robinson, Clark N. "Elementary School Children's Arithmetic Needs Arising in the Home Environment." Unpublished M.A. thesis, Stanford University, 1938.

comparison, and counting constituted about two-thirds of all the non-computational problems reported. Pupils' problems increased in difficulty as they progressed from grade one to grade six. In every grade integers and decimals made up practically all of the computations. The problems involving decimals, with but few exceptions, had to do with money transactions. Fractions were seldom used. Subtraction was used most and division was used least when the six grades were considered as a whole. Addition and multiplication were of about equal importance. Addition was most important in grade one and decreased in relative importance in the upper grades. Multiplication was needed least in grade one and became of greater relative importance in the upper grades. The need for subtraction and division varied little from grade to grade. A consideration of the processes needed in the solution of the problems reported showed the problems to be quite simple. In computations involving money in grades one and two, addition and subtraction satisfied most of the needs. Addition and subtraction combined in grades three to six accounted for about fifty per cent of the processes, multiplication for about thirty-five per cent, and division for the balance.

The author pointed out that the study was limited because it did not run through the entire year, that it did not present a fair geographic sampling of pupils of the United States, that the problems reported were too few in number to warrant other than tentative conclusions.

This is an important pioneer study in this field and will certainly be of great value to those who wish to make similar investigations.

The most recent study to be made of out-of-school number experiences of children is that by Culver.[20]

[20] Culver, Mary Marjorie, "A Study of Children's Interest in Arithmetic," *Twelfth Yearbook*, California Elementary School Principals' Association, Vol. 12 (1940), pp. 60-70.

Parents of the elementary school children were requested to write down over a period of time all the questions, problems, and comments regarding quantity which their children spontaneously expressed in the home. The study was explained and materials for recording were distributed at the meetings of the Parent Teacher Association held at the respective school buildings. In addition to these group contacts, about one hundred home calls were made to personally solicit the cooperation of parents.

In this investigation it was found that interest in time is at its height between ages of six and ten. At this same age there appears to be a steady interest in measurement centering chiefly around length, weight, and liquid measure. The subject of money begins to be important at about age six with questions centering around values of coins, counting change, and how much money it will take to buy things they desire. Beginning at age eight, there is considerable activity in planning how to earn money, and how to save allowance in order to buy things they want. From age nine, children of this sampling show interest in family income and expenses.

Fractions are not important at any age. Those fractions mentioned are halves, thirds, fourths, and eighths. Sixteenths are used in connection with music.

Counting objects is a very important activity from age five through eight. They count food, toys, piano keys, chairs, eggs, telephone poles, steps, children, etc. Much of the counting is seemingly just for fun. Two of the most frequent functional applications are to set table and to divide food.

In regard to processes, it was found that addition ranks first in frequency of use, subtraction second, multiplication third, and division fourth. Particularly from ages five through eight, there is some tendency to do abstract adding for fun. Computational puzzles seem to be enjoyed at about ages eleven and twelve.

The field of research in the out-of-school

arithmetical use by children has barely been scratched. It has potential data of inestimable value that will not only aid in determining the appropriate subject matter to be taught at the various maturity levels of the child but data which will aid in determining the socially desirable arithmetical content of the curriculum.

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A Bit of a Book

OF COURSE, I like the book I am reading. It is new. It tells about a man's life. It is all very interesting. It expresses some old fashioned ideas to which I agree. I like especially this man's testimony as to the value of his training in mathematics. I knew it all of the time, of course, but I never let on. I thought if I told you about it, then may be you would tell somebody else who would tell somebody else. Catch on? And as the author himself might have said, Let me give you some "hum-dingers."

"Drill in mathematics is valuable to one who is to become a writer, for it leads to accuracy of thinking, and accuracy of thinking is, for some kinds of writing, the foundation of everything. Given accuracy of thinking, clarity of expression can ordinarily be attained."

"I am told that mental arithmetic is not highly regarded in modern schools; it is supposed to belong to the horse-and-wagon era of education. If so, modern students are denied a valuable discipline. To stand up before others, to think on your feet, to drive through to the answer, is good training for situations that call for quick thinking; and such situations life frequently presents."

"A subject in which I did well was grammar. The subject is thought of as far distant from mathematics, yet the quality that enabled me to do well in it was, I think, the same that enabled me to do well in arithmetic and algebra. Either that, or the trait that grammar developed in me was the same that mathematics developed. In either case the quality was precision of thought."

"I do not think highly of education that consists of mere imparting of information."

These are some of the reactions of an educated man as he looks back at his early training. Somehow, I think I understand—and agree. The book is called, "The Education of an American" and the author is Mark Sullivan. Need I add that it is a mighty fine book "from kiver-to-kiver"?

WILLIAM W. STRADER

Mathematics as Applied to Apprenticeship in Trades*

By C. A. Rothe
Assistant Supervisor of Apprenticeship, Industrial Commission, Milwaukee

A plumber and for the past twenty years an assistant supervisor for apprentices for the state of Wisconsin considers it an honor and a privilege to address the nation's most brilliant and best-looking mathematics instructors on the subject of "Mathematics as applied to Apprenticeship in Trades."

Being on the program in the closing hours of the convention makes me think of the story of the speaker who was addressing a large audience and was nicely started when one of his audience shouted, "Louder please." Continuing his speech in a louder tone he was again interrupted from the very rear of the room by another "Louder, please," at which remark the first man shouted, "Sit down, you've heard too much already."

This, no doubt, is how many of you feel, or will feel, after hearing the little I have to say as I am no orator nor instructor of mathematics but one of the many who know something of the urgent need of mathematics as applied to the many trades and the need for skilled mechanics.

Many apprenticeship-conscious employers and I are sorry to state that too many of you high school instructors are training students for college only and giving too little thought to the large percentage of your students who will enter industry.

I think I am safe in saying that not in excess of 25% of your high school graduates even start or complete four years of college.

What about the 75% or the bulk of your students?

As to the 25%, many will become engineers, lawyers, bankers, accountants and teachers, whom we all know must possess a thorough knowledge of mathematics.

Let us look into the future of the 75% of your students who do not go to college. These students may or may not be honor students, but it is not always the college graduate or the honor student who makes the most money or the greatest success in life. Do you instructors realize that many apprentices, who are not college graduates, nearing the completion of their apprenticeship earn more money on an hourly or annual basis than many teachers with four or more years of college receive during the first to fifth years of teaching?

It might interest you to know what some skilled tradesmen earn on both an hourly and annual basis, but bear in mind these rates vary upward or downward according to the locality, size of city and the law of supply and demand.

In Milwaukee some of our leading trades pay as follows:

Machinist, 80¢–$1.00 per hr. or $1600–$2000 per yr.

Toolmaker, 80¢–$1.75 per hr. or $1600–$3500 per yr.

Molders, 85¢–$1.25 per hr. or $1700–$2500 per yr.

Hammersmiths or Drop Forge Man, 75¢–$1.75 per hr. or $1500–$3500 per yr.

Carpenters, $1.00–$1.50 per hr. or $1500–$2250 per yr.

Plumbers, $1.35–$2.00 per hr. or $2160–$4000 per yr.

Electricians, $1.00–$1.50 per hr. or $2000–$3000 per yr.

Sheet Metal Workers, $1.35–$1.50 per hr. or $2160–$3000 per yr.

* Paper read at the Summer meeting of the National Council of Teachers of Mathematics at Milwaukee, Wis., July, 1940.

Mason, \$1.25–\$1.50 per hr. or \$1950–\$2340 per yr.

Printers, 90¢–\$1.10 per hr. or \$1880–\$2200 per yr.

Our modern industry is not satisfied with your present mathematics instruction and frequently find it necessary to have shop or vestibule schools or compel their employees and apprentices to review or receive further training in mathematics and other subjects, which they feel are necessary in their business, in vocational schools, trade schools or through correspondence courses.

Apprenticeship supervisors, employment managers and vocational counsellors too frequently find applicants for apprenticeship either lacking the foundation of mathematics or the art of applying it. For example,—Many high school graduates coming into my office seeking apprenticeship are given this simple problem in arithmetic.

If you work for me 8 hours and 15 minutes and I agree to pay you $22\frac{1}{2}$¢ per hour, how much money would I owe you?

Too often the answer is \$185.625 or \$18.5625 instead of the correct answer of \$1.85625. Too frequently the applicant endeavors to try multiplying common fractions by decimals or is inclined to dodge the fractions and decimals entirely. A very common answer is, "*about* \$1.80," or "*about* \$1.85" or "*about* \$1.86" or any other combination of figures. This shows a lack of ability or forgetfulness in applying mathematics to a practical use.

Let us for a moment analyze the use and need for mathematics in the toolmaking and machinist trade.

Arithmetic, geometry and physics are utilized every day in these trades and the ability to use them efficiently means more money in the pay envelope. These trades utilize arithmetic every day in the use of reading the rule or scale or other precision measuring devices, addition, subtraction, division, multiplication, fractions, decimals, ratio, square and cube root, the number of degrees in a circle, capacities and other phases of the subject.

Where are these subjects used?

1. The use of the rule or scale:

Imagine a journeyman in these traces being asked to make a precision die, jig or fixture without a knowledge of the machinist scale or rule which is graduated down to 1/64 of an inch. All work requiring finer measurements than 1/64 of an inch requires the use of micrometers, dividers, plugs, or gauges which are calibrated in decimals to 1/10,000 of an inch.

2. Addition, subtraction, division, multiplication, decimals and fractions are used in laying out pieces to be machined which require constant checking.

3. Ratio is used in gear lay-out and in the machining of gears, pulleys, or the art of thread cutting.

4. Square and cube root are used in gear layout.

5. The number of degrees in a circle is used very materially in the laying out and machining of any complete circular or segmental piece.

6. Capacities of areas and solids is used in the making of dies to allow for clearances, gauge of metals in determining the cutting or punching limitations of the die.

7. Algebra is not used to any great extent by the toolmaker or machinist unless his desires, ambitions and abilities lead him to the designing, drafting, engineering or experimental departments where it is very essential in conjunction with trigonometry.

8. Geometry—

Plane and solid geometry are used in the laying out and machining departments in finding exact centers, unknown dimensions and etc.

9. Physics.—The numerous uses of the subjects given in physics are applied every minute of the day in an almost unconscious manner.

a. The law of momentum or force is manifested in the use of the fly wheel on

the auto which keeps the engine in motion until the next explosion in a cylinder.

b. Expansion and contraction. This principle is used in shrinking the heated hub of a fly wheel or pulley on to a cooled shaft.

c. Centrifugal force,—this principle is used in the making of starters for cars or the making of the modern cream separators.

d. The law of pulleys is beautifully illustrated in the transmission case of an auto, the crane or hoist.

e. The law of the inclined plane is used in the making of cam shafts for autos.

f. The law of the screw is used in the making of a thread on the lathe or worm gears and the like.

The use of mathematics in trades other than the toolmaker and machinist is just as essential and the use of it cannot be stressed too strongly on the high school mathematics instructors, as they are the ones who give the instruction to the future apprentices or the 75% of high school graduates who do not go on to college.

The mechanical, architectural or structural draftsman not only uses your arithmetic, algebra, geometry or physics, but also must acquire a knowledge of trigonometry, triangulation, logorithms, calculus and the use of the slide rule in making rapid and accurate calculations.

The carpenter uses board foot measurements in measuring his lumber and material needs and every day uses arithmetic as well as geometry in laying out stair stringers and rafters.

The sheet metal mechanic employs arithmetic and geometry on every job he lays out.

The painter must know how to figure the number of square feet in a room to enable him to determine how many gallons of paint or rolls of wallpaper will be required for the job.

These trades have in their ranks many so-called contractors, who in reality are a bunch of guessers. They guess at the amount of materials to be used, they guess at the labor costs, they guess they can make a profit, they guess at their overhead and depreciation and too often wake up to the reality of being broke.

You may say, "How sad after they have worked long hard hours and years" and "now on W. P. A., P. W. A., or relief." These guessers all know that accurate bids and costs can be had by using mathematics, common sense and a knowledge of the practical end of their respective trades.

Mathematics properly used can help them in establishing a needed and adequate bookkeeping system, material costs, labor costs, a profit or loss and the difference between a successful business and a bankruptcy.

This guessing evil among contractors can be eliminated as far as mathematics is concerned by you instructors offering a course to these guessers in night school which must be given in a practical and understandable manner.

Our Milwaukee Vocational School has students in their night classes from 16 to 60 years of age and you might be interested in knowing that there are approximately 2,000 night school students enrolled during the school year taking mathematics from arithmetic through calculus.

Other ways of eliminating future guessers are:

1. Give a good review of arithmetic in the last year of high school instead of the more pleasant athletics, band, orchestra or glee club, and spend more time on arithmetic than foreign languages and ancient history.

2. By teaching the subject in such manner as will assure you that the student understands the subject and knows how to apply it.

3. By not passing the backward student into the next grade until the subject is mastered even though mother or father does complain to the principal.

4. By making this subject more interesting by applying its use to every day problems.

5. By teaching the subject in such a manner as will assure you, the future employers, and the apprenticeship supervisors that you are doing a good job.

Mathematics is an interesting study to those who receive the necessary foundations in the early grades but it is also a jinx to those who do not understand it.

There are mathematics instructors who love teaching the subject, put their hearts into their work and impart their knowledge of the subject to the students. Others even though in possession of the necessary diplomas, certificates or sheepskins, which tells their school board that he or she is qualified to teach mathematics in reality would make much better Latin, English or history teachers due to the lack of ability to teach the subject as it should be taught.

You instructors have talked white collar jobs, college education to the point where we Americans are facing a surplus of this kind of worker as is manifested by 10% of the engineers, doctors and lawyers doing 90% of the work in these fields while the other 90% of these workers are making a bare living at their professions.

The teaching profession is over-stocked to the point where you have too large a substitute teacher list, resulting in a bare existence wage for new teachers.

This unemployment condition is not prevalent in the trades and especially in Milwaukee where apprenticeship and mathematics are given their proper amount of attention.

At present there are about 400 orders in the U. S. Employment Office asking for skilled machinists, toolmakers, patternmakers, molders, heat treaters and draftsment at rates from 85¢ to $1.75 per hour.

The federal government through its federal committee on apprentice training is promoting apprenticeship for our American youth to industry and educational groups. We all realize that our source of mechanics coming from Europe is no more, due to the limited number permitted into the country and the present war.

Tomorrow is the 4th of July and on the 5th we return to our preparedness program short of skilled mechanics at good wages while thousands pound the pavements seeking employment.

A preparadness program will need students well versed in mathematics, who possess a desire for a trade, the physique and the real desire to work.

Hitler built his war machines with the aid of skilled mechanics.

In closing may I ask, will you mathematics instructors put your shoulders to the wheel and give the 75% who do not go on to college the necessary mathematics to become the skilled mechanics to build the planes, the boats, the guns, ammunition and everything necessary to keep America a democracy?

Problem

Mrs. A., Mrs. B. and Mrs. C. and their three daughers each bought cloth and lace. Each of the six bought as many yards as she paid cents per yard and each daughter paid 63¢ less than her mother. Jane bought 23 yards less than Mrs. A.; Elizabeth bought 11 yards less than Mrs. B.; the other daughter was named Ann. Whose daughter was each girl?—Contributed by J. C. Brown, Supt. of Schools, Pelham, N. Y.

◆ THE ART OF TEACHING ◆

Correlation of Solid Geometry with Other Subjects

By Sophia R. Refior
Scott High School, Toledo, Ohio

We are faced with the same problem which confronts most public schools, namely of adapting our teaching to pupils of varying ability. Some students are well prepared and gifted mentally so they can readily visualize the various figures, while others seem to have only a minimum of imagination. This disparagement is evident in Plane Geometry but is more apparent in Solid Geometry. Three-dimensional figures on two-dimensional surfaces are conundrums which remain mysteries. These pupils who lack the power of imagination and abstraction often have mechanical ability. They wish to enter professions where they can use their deftness and they realize their need for more of the theories of solid geometry.

To aid this group, each member is obliged to make a model of a theorem of solid geometry. Some models are made of stiff yellow paper and are done in connection with their mechanical drawing. These are usually very neat and are worthy of being placed in our exhibition case. This type was used for the proposition for the lateral area of a regular pyramid and for the congruency of two trihedral angles. For the latter theorem, the lettering was put on both sides of the paper so it could be reversed and thus show the difference between "congruent" and "symmetric." To illustrate the smaller triangles in three other planes an additional model was made for the interior. It was interesting to watch the progress of the class. The first day of the assignment, with only flat surface drawings, just one-half of the class could recite—the others had made an effort, but were mystified. The solids were explained step by step and then passed around the class who examined them with great eagerness. Everyone was enthusiastic, their bewilderment gave way to understanding and appreciation. The following day the class recited 100%.

The diagonals of cubes, parallelopipeds and the diagonals of their faces were illustrated by wire skeletons; spherical surfaces by tennis balls cut open and fitted out by wire triangles and arcs. Volumes of prisms, spheres and pyramids were made out of solid wood in the work shops.

Each pupil was obliged to make at least one contribution. Their enthusiasm carried them to making as many as ten objects. Some were very crude but were accepted if they illustrated a geometric principle. A piece of cardboard, a pencil and string were used for "If a line is perpendicular to two intersecting lines, it is perpendicular to the plane of these lines."

The Platonic Solids were studied the first week in December with varying degrees of success. Then the assignment was again made. "Each pupil will bring one of these regular polyhedrons," with the suggestion that he may use fancy paper or other decorative material. The results were astonishing and delightful to the eye as well as to one's geometric sense. The students in art used pastel shades and then dipped the solids in silver dust. A committee was appointed who purchased a Christmas tree. Some pupils put a few buttons inside of the solids and gold cord in one end. These were then hung upon the tree which became a thing of beauty with ornaments of futuristic design. After a few days of enjoyment the tree and its contents were given to the Day Nursery.

EDITORIAL

The Problem of Teaching Gifted Pupils

THE FOLLOWING quotation from the report of the "White House Conference on Child Health and Protection" in 1931 is especially significant today:

> It is agreed that in a democracy more than in any other form of government, high grade leadership is essential. The United States of America with its Congress; with its forty-eight commonwealths, each with its legislature; with its hundreds of municipalities, each with its own local government; must have intelligent leaders or fail in the struggle. Surely there was never greater need of able leadership than at the present time. And yet there are one million and a half children in our public schools with exceptionally good brains and exceptionally high intelligence, who need only the permission and the opportunity to develop the leadership for which they have the foundation; therefore, we urge that the White House Conference and all intelligent, patriotic citizens of the United States take active and efficient steps to save this large number of children from the idleness, the more or less malicious mischief and the neglect which is their portion in the average public schools of today. Aside from the injustice to the child himself, it is almost a social crime to neglect these highly endowed children.

In the emergency which we are now entering training for leadership in our democracy is of supreme importance. In fact our very existence may depend upon the kind of leadership the country may have in the years ahead.

Naturally, in a democracy like ours we need also an intelligent type of followership which we hope may come from the less gifted or able pupils. But this is more likely to happen if the kind of leadership we have is of the right kind. The most retarded pupil in the secondary school today is the pupil of scholarly mind and it is our duty to study the situation most carefully in the days ahead.

The September 1941 Research Bulletin (Volume XIX, No. 4) of the National Education Association on "High School Methods with Superior Students" will be of great interest to teachers everywhere. It is published by the Research Division of the National Education Association, 1201 Sixteenth Street, Washington, D. C., and can be secured for twenty-five cents postpaid if remittance accompanies order. There is a ten per cent reduction on two to nine copies, a twenty-five per cent reduction on ten to ninty-nine copies, and a thirty-three and one third per cent reduction on one hundred or more copies.

W.D.R.

NEWS NOTES

Dr. David D. Leib, 61, member of the Connecticut college faculty and administrative staff, who as professor of mathematics, registrar, and director of admissions had served the college devotedly for nearly 25 years, died suddenly at the college last night before the commencement exercises in which his daughter Harriet-Ellen was to receive her degree.

Each year the Honorary Mathematics Club of the New Jersey State Teachers College at Montclair gives a year's subscription to THE MATHEMATICS TEACHER to that person who achieves the second highest rating in Mathematics for the four undergraduate years. The TEACHER goes this year to

Miss Shirley Stamer
47 Monticello Avenue
Newark, New Jersey

Professor Howard Fehr of Montclair thinks that "Perhaps a news notice of such a prize might inspire other Teachers Colleges to award the same prize and thus help build our organization."

The Students and Teachers of the State Teachers College at Carbondale, Illinois, held their third "Annual Mathematics Field Day" on February 15, 1941.

The first meeting of the 1941–42 season of the Men's Mathematics Club of Chicago featured two speakers who presented topics of current interest to the teacher of mathematics or science.

Mr. James A. Longman, teacher of physics at Lane Technical High School, spoke on the general topic, "The Relationship of Mathematics to Physics in the High School." Mr. Longman has been associated with Mr. James P. Coyle, one of the authors of the Millikan, Gale, and Coyle text in high school physics.

Professor Merrill P. Gamet, Assistant Professor of Civil Engineering and Registrar of the Northwestern Technological Institute, gave concrete suggestions on, "What the Engineering School Desires from Its Entering Students."

At previous meetings of the year the following programs were rendered:

On March 21, Dr. Wm. H. Erskine of Wright Junior College spoke on Sculpture and Mathematics." On April 18, Joseph A. Nyberg of Hyde Park High School discussed the "Fifteenth Yearbook of the National Council of Teachers of Mathematics" and Dr. H. T. Davis of Northwestern University spoke on "The Mathematical Theory of History" and on May 16 Marx Ernest Holt of the Fiske School spoke on "Mathematics for the Tired Business Man."

The Sixteenth Annual Conference of Teachers of Mathematics was held at Iowa City, Iowa, on October 10 and 11, 1941. The program follows:

Friday Morning, October 10, 1941
ROSCOE WOODS, *presiding*

10:00 A.M. Address: Opportunities in mathematics that can be offered to the good student: Part I. Suggestions for a six-year program. Virgil S. Mallory, New Jersey State Teachers College, Montclair.

10:30 A.M. Address: Concepts for students in engineering. F. M. Dawson, Dean of College of Engineering, University of Iowa.

11:00 A.M. Address: Making capital of opportunity. LaRue Sowers, Ottumwa High School. Discussion

Friday Afternoon, October 10, 1941
E. N. OBERG, *presiding*

1:30 P.M. Address: What mathematics should do for the brighter pupil. Beulah I. Shoesmith, Hyde Park High School, Chicago, Illinois.

2:00 P.M. Address: My responsibility as an American teacher. Mrs. Mary Ethel Pomeroy, Abraham Lincoln High School, Council Bluffs.

2:30 P.M. Address: The mathematics problem in Australia. Thomas H. Roberts, University of Iowa.
Discussion

Friday Evening, October 10, 1941

6:00 P.M. Conference Dinner. Play by University High School Students.

Saturday Morning, October 11, 1941
L. E. WARD, *presiding*

9:30 A.M. Address: An ounce of prevention. Miss Shoesmith.

10:00 A.M. Address: Opportunities in mathematics that can be offered to a good student: Part II: Illustrative Materials. Professor Mallory.

10:30 A.M. Address. The need of mathematics in chemistry. G. Glockler, Professor and Head Department of Chemistry, University of Iowa.
Discussion

Professor F. L. Wren of George Peabody College for Teachers, Nashville, Tenn., was the speaker at the meeting of the Mathematics Section of the Western Pennsylvania Education Conference on October 11, 1941, at the University of Pittsburgh.

Officers of the Mathematics Section:

Chairman: Dr. Karl H. Stahl, State Teachers College, California, Pa.

Vice Chairman: Dr. W. J. Wagner, Taylor Allderdice H.S., Pittsburgh, Pa.

Secy.-Treas.: Miss Ida M. Price, Avalon H.S., Avalon, Pa.

Professor Wren's topic: Evaluation of Mathematics in Secondary Education.

KARL H. STAHL

Among the fifty world scholars to receive honorary degrees from the University of Chicago at its Fiftieth Anniversary Jubilee was George David Birkhoff, professor of mathematics at Harvard University.

The Kentucky chapter of the National Council of Teachers of Mathematics and the Kentucky chapter of The Mathematics Association of America held their second annual joint meeting in Lexington on Saturday, October 25 with approximately one hundred in attendance. The purpose of these joint meetings is to foster understanding of the problems of each group and to bring a closer cooperation between the teachers of high school mathematics and the teachers of collegiate mathematics.

The group convened at 9:45 a.m., Miss Tryphena Howard, president of the Kentucky

chapter of the Council, presiding. The following program was presented.

"Some Psychological Aspects of the Teaching of Arithmetic," Mr. M. E. Schell, Western Kentucky State Teachers College.

"Errors on Freshman Entrance Examinations," Dr. H. A. Wright, Transylvania College.

"Mathematics Enters Naturally into Practical Problems," Dr. O. T. Koppius, Department of Physics, University of Ky.

"Mathematics and Science in the National Defense Program," Professor D. W. Pugsley, Berea College.

"Mathematics and the Environment," Sister M. Raymond, Ph.D., Ursuline College, Louisville.

"On the Invariants of a Conic Section," Dr. C. G. Latimer, University of Kentucky.

"Mathematical Instruments," Miss Edith Wood Okolona High School, Route 4, Louisville.

"Treatment of a Certain Improper Integral," Dr. W. C. Wineland, Morehead State Teachers College.

Following the formal program, an informal luncheon meeting of the groups was presided over by L. A. Fair, president of the Kentucky chapter of the Association.

Tryphena Howard

The Colorado Branch of the National Council of Teachers of Mathematics held its annual meeting on Oct. 23, 1941. The program follows:

Business meeting and election of officers.

Delegate's report of the Atlantic City meeting —H. W. Charlesworth, Denver.

"Explaining to the Joneses"—Dr. A. W. Recht, University of Denver.

President, Miss Alfhild Alenius, Denver.

Vice President, A. E. Mallory, Greeley.

Secretary, Valworth R. Plumb, Denver.

The Tulsa Council of Teachers of Mathematics met in October for tea and a program. Miss Copeland presiding at the tea table was assisted by two Central High School girls.

The program consited of a talk on geometric Design used in Flower Arranging by Mrs. Charles H. Haralson. Mrs. Haralson is considered an authority on flower arranging, having attended several schools on this subject. The teachers were quite fascinated to see how their stand-bys the isosceles, the equilateral and the scaline triangle, symmetry, proportion, and the circle influenced the arrangement of Mrs. Haralson's lovely bouquets.

Officers of T.C.T.M.

President—Ware Marsden

Vice-Pres.—Eunice Lewis

Secy.-Treas.—Marguerite Randall

One of Europe's most brilliant engineers is safely in Providence, R. I., today, a refugee from Nazi Germany who is ready to begin a new life as professor of engineering at Brown University.

He is Dr. Willy Prager, thirty-eight-year-old expert in applied mechanics. Dr. Prager's contributions to knowledge of vibrations, the strength of materials, the theory of structures, elasticity and dynamics have revolutionized many phases of engineering, including the design of airplanes and other instruments of war.

Before Hitler's armies marched, Dr. Prager was acting director of the Institute of Applied Mechanics of the University of Göttingen, professor at the Technical University of Karlsruhe, structural inspector for the German Airsport League and scientific adviser to the Fiesler Aircraft Company, one of Germany's largest plane manufacturers.

He was also a special adviser to the British Air Ministry. After being "requested" to leave his native country in 1933, Dr. Prager accepted a position as professor of applied mechanics at the University of Istanbul, and continued to aid British air interests through his research and his technical advice. Recently he was made an associate fellow of the Institute of the Aeronautical Sciences, New York.

The fall meeting of the Mathematics Section of the Colorado Education Association was held on October 23 at Denver. The following program was given:

"Mathematics the Dynamic Way" by Arthur Norman Jackson.

"Democratizing Mathematics" by Nathan Setshiller Court.

The officers of the section are:

Alfhild Alenius—President—545 Washington Street, Denver.

A. E. Mallory—Vice-President, Colo. State College of Education, Greeley.

Valworth Plumb—Secretary-Treasurer, 4763 Raleigh Street, Denver.

The Members of the Executive Council are:

Term expires November 1941

R. I. Ashbaugh	Longmont
Lillian Duer	Denver
Claribel Kendall	Boulder
Wendell I. Wolf	Denver

Term expires November 1942

Florence Barnard	Aurora
Mary Doremus	Denver
J. C. Fitterer	Golden
Courtland Washburn	Erie

Term expires November 1943

Dwight Gunder	Fort Collins
Ernst Kruse	Greeley

Margaret McGinley	Denver
Frances Smith	Sterling

Dr. Gilbert A. Bliss, Martin A. Ryerson distinguished service professor and chairman of the department of mathematics leader in the field of mathematical analysis and civilian ballistics aide during the first World War at the University of Chicago retired from Active service at the age of 65 years.

A member of the University faculty for 33 years, Dr. Bliss was born in Chicago in 1876. He was awarded the Bachelor's and Master's degree in 1897, and the Ph.D. degree in 1900 at the University of Chicago, and an honorary Doctor of Science degree at the University of Wisconsin in 1935. After teaching at the universities of Minnesota, Missouri, and Princeton, he was appointed associate professor of mathematics at the University of Chicago in 1908, full professor in 1913, chairman of the department in 1927, and Martin A. Ryerson distinguished service professor in 1933.

In addition to his research in mathematical analysis, Dr. Bliss conducts a course in exterior ballistics—the science which sends long-range shells truly to their mark. He was scientific expert of the range firing section of the U. S. Army in 1918. His published works include books on the calculus of variations, and the science of exterior ballistics. A member of many learned societies, both here and abroad, Dr. Bliss is past president of the American Mathematical Society.

Metropolitan New York Section Mathematical Association of America

The Board of Governors of the Mathematical Association of America has recently approved the establishment of the Metropolitan New York Section, the 24th section of the Association, which held its organization meeting last April at Queens College.

A set of by-laws for our section was also approved by the Board of Governors. One paragraph of the by-laws reads as follows:

"There shall be an executive committee composed of the officers, a representative of each collegiate department of mathematics in the metropolitan area which accepts an invitation to name a representative, a representative of each association of high school mathematics teachers in the metropolitan area, and a representative of science and industry to be selected by the officers."

The following persons are officers:

T. Freeman Cope, Queens College, Chairman

John A. Swenson, Andrew Jackson High School Vice-Chairman.

Howard E. Wahlert, N. Y. University, Secretary.

Frederic H. Miller, Cooper Union, Treasurer.

W. D. Reeve, professor of mathematics at Teachers College, Columbia University was a speaker at the High School Section and also the Mathematics Section of the Tennessee State Teacher's Association meeting at Knoxville in October.

Mathematics of the Detroit Mathematics Club

It Multiplies $\times$ *your friends*

It Divides $\div$ *your worries*

It Adds $+$ *to your pleasures*

It Subtracts $-$ *from your discomfort*

(*Join the Detroit Math. Club!*)

OFFICAL PROGRAM

1941–42

First Meeting

(Joint Meeting with M.E.A. Math Sction)

Friday, October 24, 1941 Fort Shelby Hotel

Topic: "What the High Schools Ought To Teach"

Speaker: David M. Trout, Central Michigan College of Education

Luncheon—12:15 P.M.

Second Meeting

Thursday, December 4, 1941 Post Intermediate

Illustrated Lecture: "Motion Picture for the Teaching of Mathematics"

Speaker: Henry W. Syer, Culver Military Academy

Tea 3:15 Meeting 4:00 P.M. Sharp

Third Meeting

Thursday, February 12, 1942 Cass Tech. High School

Topic: "Meeting the Mathematical Needs of Technical Training"

Speakers: Cass High School Mathematics Faculty, Harry M. Keal, Head of Math. Dept., Presiding

Tea 3:15 Meeting 4:00 P.M. Sharp.

Fourth Meeting

Thursday, March 26, 1942 Pershing High School

Topic: "The Cleveland Experiment in Mathematics for Everyday Life"

Speaker: Joseph M. Jacobs, East High School, Cleveland, Ohio

Tea 3:15 Meeting 4:00 P.M. Sharp

Fifth Meeting

Thursday, May 7, 1942

McMichael Intermediate

Topic: "Growth, Attitude and Achievement in Mathematics," (Illustrated with slides)
Speaker: Dr. Willard Olson, University Elementary School, University of Michigan
Tea 3:15 Meeting 4:00 P.M. Sharp

Officers of the Club

Duncan Pirie	Jefferson Int.	President
Bernhard Pagel	Nolan Int.	Vice President
Laura Crone	Durfee Int.	Secretary
Robert Lankton	Northeastern	Treasurer

FALL MEETING OF THE MATHEMATICS SECTION, BAY SECTION, C. T. A.

The fall meeting of the Mathematics Section of the Bay Section of the California Teachers' Association was held December 6th at International House in Berkeley. Trying an innovation, the Chairman in charge, Mrs. Maud O. Volandri planned a "brunch" at 10:15, after which the program was held. It was a success in every way. There was a large attendance from the entire Bay Section, and the program was enthusiastically received. Professor Peter L. Spencer of Claremont Colleges, Pomona, was the speaker. His topic was "Let Us Consider Our High School and College Students' Preparation for Mathematics." This was followed by interested discussion.

Professor Spencer, Miss Hesse, and Mr. Winegardner spoke on various aspects of the meeting of the National Council to be held in San Francisco in February. Mr. Winegardner announced that the Discussion Luncheon on February 21st will be attended by the Bay Section members and be counted as our regular spring meeting.

The Mathematics Section of the High School Conference (the Illinois Association of the National Council of Teachers of Mathematics) met on November 7, 1941 in the McKinley Memorial Church, Champaign, Illinois.

The Chairman, Miss Anice Seybold, Monticello, Illinois, called the meeting to order at 9:15 and introduced the first speaker of the morning session, Captain C. L. Wood of the Air Corps Technical School, Chanute Field, Rantoul. He spoke on "The Placement and Training of Students in the Air Corps Technical Training Schools."

Capt. Wood opened with, "I am speaking as a member of a system for which your schools are preparing students." He went on to describe the series of tests (one of them was a Mathematics achievement test) given every student in order to place them in the field for which they were best fitted. He explained the different types of instruction offered in each of the different training schools. In doing this he made you see the necessity for a sound foundation in mathematics.

Mr. E. B. Miller, Head of Mathematics Department, Illinois College, Jacksonville, next addressed us on "Some Results of a Placement Test in Arithmetic and Algebra." He passed out mimeographed sheets analyzing the results of the algebra part of a test given to over 43,000 students in Kansas. Only one question was answered correctly by as many as half of the students. The rest ranged on down to only 2.1% correct. Mr. Miller claimed that these results should be a challenge to the teacher to improve instruction and not lower standards.

After a short intermission the following announcements were made:

1. Mr. Schreiber urged more teachers to join the National Council and to buy their yearbooks.
2. An announcement concerning the meeting of the Central Association of Science and Mathematics Teachers in Chicago was made.
3. Dr. Hartley told us how to find the room where the Mathematics Luncheon was to be held.
4. The following nominating committee was appointed: Mr. Clark, Champaign, Miss Wilkins, Danville, and Miss Innes, Dundee.

Miss Shoesmith, Hyde Park High School, Chicago, then addressed us in a very interesting and witty manner upon "What Mathematics Should Do for the Brighter Pupil." She stressed that the emphasis that has been placed upon the less able pupil has resulted in serious neglect of the brighter pupil. The superior pupil should be given an enriched course which spreads out the base instead of being pushed ahead to higher subjects.

The meeting was then adjourned until 2 P.M. and everyone was invited to visit the exhibit in the basement.

The fourth Annual Mathematics Luncheon assembled in Room 314 North, Illini Union Building. There were 127 present and many were turned away. Miss Seybold introduced Dr. Hartley, Chairman for luncheon arrangements, who very cleverly described some of the difficulties to be overcome in arranging for such an occasion. Following the serving of the meal Miss Hildebrandt, Proviso Township High School, Maywood, was introduced. She gave a brief talk on "Some Visual Aids in the Teaching of High School Mathematics." She supplemented her remarks with models made by some of her students.

The afternoon session was opened with the

reading of the minutes of our meeting in 1940.

Mr. Clark then gave the report of the nominating committee: Mr. Walter W. Willis of Dupo for Chairman, Mr. Walter Barczewski of Waukegan for Vice-Chairman, and Miss Lois Busby of Danville for Secretary. Since there were no further nominations, the secretary was instructed to cast a unanimous ballot.

Miss Seybold then introduced Mr. Elvin S. Warrick, Mathematics Librarian, University of Illinois, who illustrated his lecture "Browsing Among Early American Arithmetics" with many interesting photographs of pages from very old books. From these one could follow the progress of arithmetic. Some of the ideas present in these old books have long since been discarded, but some are being revived today as "up-to-date" innovations.

Carrie Belle Abbott, Havana Public Schools, then outlined "A Modern Arithmetic Curriculum for Grades One to Eight." She gave a detailed outline of a progressive arithmetic curriculum.

Rueben Baumgartner, Freeport High School, was introduced next. He explained the method of "Classification of Ninth Grade Pupils" used at Freeport. He had statistically worked out an alignment chart to weigh the different elements involved. The three main criterion used were:

(1) Grades on Lee's Prognosis Algebra Test,
(2) Grades in eighth grade arithmetic, and
(3) I.Q. scores.

Gertrude Hendrix, Eastern Illinois State Teachers' College, Charleston, in speaking on "Freedom to do Versus Right to Do" gave us a modern and alive method of approaching and putting across the idea of independent and independent variables. For example, if you are constructing a triangle, (1) how many choices have you for the first angle? (2) How many choices have you for the second angle? (3) Now how many choices have you left for the third angle? In closing she claimed that this method of teaching would give us some very important by-products in developing ideals in the child such as sportsmanship, emotional maturity, etc.

Blanche Raman, Lanphier High School, Springfield, whose topic was "Technical Mathematics for Junior and Seniors," described an interesting experiment she was carrying out. This course was designed for students who had not necessarily majored in mathematics. Her emphasis throughout was in solving practical problems—not in proofs. Material was gathered from a score of books on Shop Mathematics, Vocational Mathematics, Mathematics for Engineers, Mathematics for Aviators, etc. This course has been so popular that it is being extended into a full year instead of just one semester as it was originally planned.

The remainder of the afternoon session was devoted to a discussion led by Miss Hendrix. Many people entered into the discussion with the result that it was very profitable.

The attendance for the afternoon session was not quite up to the 425 that attended the morning session.

W. W. Willis, *Secretary*

The Association of Teachers of Mathematics in New England held its thirty-ninth annual meeting on Saturday, December 6, 1941, at Boston University. The program follows:

Morning Session

10:00 Social Period and Business Meeting.
10:15 "Geometric analogy a source of discovery." Professor George Polya, Brown University.
10:50 "Constructions with the parallel ruler." Mr. M. Philbrick Bridgess, Roxbury Latin School.
11:15 Public forum on topics of geometry for the secondary school:
- (i) Objectives of a geometry course.
- (ii) Use of postulates.
- (iii) Meaning of "line," "angle," "parallel."
- (iv) Use of directed quantities.

12:30 Luncheon at Hotel Lenox, Boylston and Exeter Streets. (*No reservations necessary.*)

Afternoon Session

2:00 "Remarks concerning the teaching of solid geometry." Professor Dirk J. Struik, Massachusetts Institute of Technology.

Reserve these dates for future meetings in 1942:
January 17, and April 11, Dinner Meetings.
March 7, Winter Meeting, Worcester.
May 2, Spring Meeting, Boston.

Council for 1941

Professor Albert A. Bennett........President
Brown University, Providence, R. I.
Mr. John A. Marsh............Vice-President
High School of Commerce, Boston, Mass.
Mr. Harold B. Garland...Secretary-Treasurer
129 Houston Avenue, Milton, Mass.

Term Expires 1941

Miss Celia A. Russell
Weeks Junior High School, Newton, Mass.
Mr. Winfield M. Sides
Phillips Academy, Andover, Mass.

Term Expires 1942

Miss Margaret Cochran
Senior High School, Somerville, Mass.
Prof. Joseph Spear
Northeastern University, Boston, Mass.

THE Mathematics Department of Teachers College, Columbia University, will offer two new courses, relating to military affairs and defense matters, next semester, beginning February 2, 1942. These courses are intended primarily for teachers of mathematics and science who wish to make their high school courses of more practical value. The first will be given by Professor C. N. Shuster and will relate to Elementary Military Engineering and Air Navigation and will prepare one to take the examination given by the Civil Aeronautics Authority. The second will be given by Mr. G. R. Mirick and will relate to the Elementary Theory of Ballistics. These courses are also intended to serve those who wish to review parts of mathematics and mechanics and who may later have an opportunity to use them in a practical situation.

Program of the Twenty-Third Annual Meeting of the National Council of Teachers of Mathematics

February 20–21, 1942, Palace Hotel, San Francisco, California

Theme: Mathematics and Vocational Guidance Program

Friday, February 20, 1942

10:00 A.M. *Junior High School Section—Comstock Room*

Presiding: Edith Mossman, Berkeley High School, Berkeley
A Junior High School Teacher Discusses Teaching
Hildegarde Beck, McMichael Intermediate School, Detroit, Michigan
What the Schools Should Do to Further Our Defense Program
E. A. Bond, Western Washington College of Education, Bellingham, Washington
Some Important Changes in the Mathematics of Grades Seven, Eight, and Nine During the Last Quarter of a Century
Raleigh Schorling, University of Michigan, Ann Arbor, Michigan
Discussion

10:00 A.M. *Training Prospective Teachers—Tapestry Room*

Presiding: George A. Rice, University of California, Berkeley
Should Mathematics Courses for Prospective High School Teachers be Unique
Robert L. Morton, Ohio University, Athens, Ohio
The Program for the Training of Teachers at the University of California
James W. Hoge, University of California, Berkeley
The Role of Mathematics Teachers in National Defense
F. L. Griffin, Reed College, Portland, Oregon
Discussion

10:00 A.M. *Theory of Multi-sensory Aids—California Room*

Presiding: Allen R. Congdon, University of Nebraska, Lincoln, Nebraska
Mathematics in the Seattle Schools as Told in Pictures
Edith Sifton, Seattle, Washington
The Making and Use of Slides for the Teaching of Mathematics
Kate Bell, Lewis and Clark High School, Spokane, Washington
The Geometry Teacher Becomes a Motion Picture Director
Rachel P. Kenniston and Jean Tully, Stockton, California

2:30 P.M. *Arithmetic Section—California Room*

Topic: Problem with Arithmetic Instruction at the Elementary School Level
Presiding: John T. Johnson, Chicago Normal College, Chicago, Illinois
Arithmetic as Concept Building (Emphasis on Early Grades)
Marguerite Nordahl, San Diego State College
Practical Procedure for Motivation
Charles Grover, Principal Glenview Elementary School, Oakland
A Desirable Balance Between Social Arithmetic and a Science of Arithmetic
E. A. Bond, Western Washington College of Education, Bellingham, Washington
Significant Principles of Arithmetic Instruction
Frederick Breed, University of Chicago, Chicago, Illinois
Discussion

2:30 P.M. *Senior High School—Comstock Room*

Presiding: Kate Bell, Lewis and Clark High School, Spokane, Washington
Geometry for Modern Youth
Harold D. Aten, Fremont Senior High School, Oakland

A Functional Mathematics Program for Twelfth Graders Who Have Had Algebra and Geometry
Earl Murray, High School Curriculum Coordinator, Santa Barbara
Training in Mathematics Essential for Life
Allen R. Congdon, University of Nebraska, Lincoln, Nebraska
Discussion

2:30 P.M. Meeting Delegates, State Representatives, Directors—Tapestry Room

4:30 P.M. Tea, given by Bay Section Mathematics Section to Council members and guests.

8:00 P.M. General Meeting—Gold Ballroom
Presiding: Mary A. Potter, Supervisor of Mathematics, Racine, Wisconsin
Address of Welcome
Frank B. Lindsay, Assistant Chief, Division of Secondary Education, State Department of Education, Sacramento
The Effects of the Vocational Aim Upon the Teaching of Mathematics
Earle R. Hedrick, Vice-President and Provost, University of California, Los Angeles

Saturday, February 21, 1942

8:30 A.M. Annual Business Meeting—Room A

9:30 A.M. *Junior-Senior High School Section—Comstock Room*
What Can the Teacher of Mathematics Do About Vocational Guidance?
Harriet Welch, Lowell High School, San Francisco
The Need for Guiding Students in Mathematics Necessary for Defense
Charles A. Stone, De Paul University, Chicago
Guiding Students in the Choice of Mathematics
Helen J. Hunt, Vice-Principal, Claremont Junior High School, Oakland
Subject to be announced
P. B. Badger, Assistant Superintendent of Schools, Portland, Oregon
Discussion

9:30 A.M. Training Mathematics Teachers in Service—Tapestry Room
Presiding: James W. Hoge, University of California, Berkeley
In-Service Training Through Curriculum Instruction
Lesta Hoel, Supervisor of Mathematics, Portland, Oregon
In-Service Training in a Large High School
Martha Hildebrandt, Proviso Township High School, Maywood, Illinois
The Increasing Load as Concerns Securing Competence in Computation Placed upon the Junior High School Teacher
Raleigh Schorling, University of Michigan, Ann Arbor, Michigan
The University as a Reservoir for Cooperation
Griffith C. Evans, University of California, Berkeley
Discussion

9:30 A.M. *Multi-sensory Aids Demonstration—California Room*

I. Film Strips

1. Mathematical Instruments
2. The History of the Measurement of Length

II. British Mathematical Films

1. Theorem of Pythagoras
2. Angle Sum of a Triangle
3. Levers

III. Sound Films

1. Stereoscopic Mapping from the Air
2. Geometry in Action
3. Rectilinear Coordinates
4. Precisely So

12:00 Discussion Luncheon—Concert Room

It is most important to make luncheon reservations in advance stating first, second, and third choice of tables. Reservations should be made with Mrs. Ruth Sumner, Oakland High School, Oakland, California

Tables	*Leaders*	*Topics*
1.	Hildegarde Beck Detroit, Michigan	Your Problems—Our Problems—Junior High School Teaching
2.	Kate Bell Spokane, Washington	Visual Aids for Mathematics
3.	E. A. Bond Bellingham, Washington	Subject to be announced
4.	Homer G. Cain Pomona	Teaching Pre-Engineering Mathematics in Three Years, Grades 10–12
5.	Allan R. Congdon Lincoln, Nebraska	Desirable Outcomes from High School Mathematics
6.	E. C. Goldsworthy Berkeley	Mathematics in National Defense
7.	E. R. Hedrick Los Angeles	Refresher Courses for Workers in Defense Industries and for Prospective Officers
8.	Luella Holman Oakland	Mathematics, a Tool in Modern Art
9.	John T. Johnson Chicago	What Can Be Done for the Slow-moving Ninth Grade Student
10.	Lucian B. Kinney Stanford University	Mathematics for General Education
11.	Florence Krieger Rapid City, S. D.	Night School Mathematics Course for Boys, Preparatory for Defense Jobs
12.	F. B. Lindsay State Department of Education, Sacramento	Human Values in Mathematics
13.	Robert L. Morton Athens, Ohio	Systematic Instruction in Arithmetic in the Primary Grades
14.	Sara B. F. Rabourne Fresno	Problems in Senior High School Mathematics
15.	Raleigh Schorling Ann Arbor, Michigan	An Elective Course in Social Mathematics for the Late Years of the Senior High School
16.	Peter L. Spencer Claremont	How Can Elementary School Arithmetic Experience Be Oriented to Produce Functional Arithmetic Concepts
17.	M. Van Wagen and H. Colwell Yuba City	How Can General Mathematics in the Ninth Grade Be Taught More Functionally

2:30 P.M. *Arithmetic Section—Room A*

Topic: Problem with Arithmetic Instruction at the Elementary Secondary Level

Presiding: Peter L. Spencer, Claremont Colleges, Claremont

Mathematics in General Education
R. L. Morton, Ohio University

Arithmetic Needs in Secondary School Training
Frank Lindsay, Asst. Chief Division of Secondary Education, California State Department

Mathematics Practices in Secondary Schools
C. C. Trillingham, Chairman, Sub-Committee on Secondary School Practices, California Association of Secondary School Principals

Mathematics for All
Harl Douglass, Dean of the College of Education, University of Colorado

2:30 P.M. *Defense Program—Comstock Room*
Presiding: Harold M. Bacon, Stanford University
Mathematics in Present Day Industry
J. Kadushin, Education Department Industrial Relations Office, Lockheed Aircraft Corporation, Burbank
Mathematics in Our Schools and Its Contribution to Defense
Sophia H. Levy, University of California, Berkeley
Mathematics in the Naval and Military Sciences
F. Grant Marsh, Lieutenant Commander, U.S.N., Associate Professor of Naval Science and Tactics, University of California, Berkeley
Discussion

2:30 P.M. *Junior College Section—California Room*
Panel Discussion: Education for Service
Leader: L. J. Adams, Junior College, Santa Monica
Speakers to be announced

6:30 P.M. Banquet—French Parlor

Local Committees

The Co-ordinating Committee

Chairman: Miss Emma Hesse, University High School, Oakland
Advisors: Professor Sophia Levy, University of California
Professor H. M. Bacon, Stanford University
Publicity: Dr. W. H. Myers, San Jose State College, San Jose
Exhibits: Mr. Ivan C. Barker, Lowell High School, San Francisco
Banquet: Miss Mary K. McBride, Lowell High School, San Francisco
Luncheon: Mrs. Ruth G. Sumner, Oakland High School, Oakland
Hospitality: Mr. J. H. Winegardner, Piedmont High School, Piedmont
Hotel Arrangements: Mr. Leland S. Russell, Lafayette

Local Members of the Program Committees

Arithmetic: Professor Peter Spencer, Claremont Colleges
Secondary: Miss Mary Adele Newcomer, San Rafael High School, San Rafael
Junior High School: Miss Edith Mossman, Berkeley High School, Berkeley
Teacher Training: Mr. James W. Hoge, University of California
Visual Aids: Mr. Harold D. Aten, Fremont High School, Oakland
Defense: Professor H. M. Bacon, Stanford University

◆ IN OTHER PERIODICALS ◆

By Nathan Lazar
The Bronx High School of Science, New York City

The American Mathematical Monthly

October, 1941, Vol. 48, No. 8

1. Allen, E. S., "The Scientific Work of Vito Volterra," pp. 516–519.
2. Copeland, A. H., "Fundamental Concepts of the Theory of Probability," pp. 522–530.
3. Wong, Yung-Chow, "Some Properties of the Triangle," pp. 530–535.
4. Mehr, Emanuel, "The Geometry of the Triangle in the Kasner Plane," pp. 535–539.
5. Fertig, R. A., "Graphical Solution of Simultaneous Equations of the Fourth Order," p. 540.
6. Thèbault, V., "Bibliography on Cyclic Properties of Miquel Polygons," p. 541.
7. Mancill, J. D. "On the Solutions of a Certain Class of Partial Differential Equations," pp. 541–542.
8. Golubew, W., "On the Representations of Numbers as Sums of Figurate Numbers," pp. 543–544.
9. Burton, W. W., and Miller, W. G., "The Integral of Powers of the Secant and Cosecant," pp. 544–545.

Bulletin of the Kansas Association of Teachers of Mathematics

February, 1941, Volume 15, No. 3

1. Olds, Edwin G., "The Use of Applications for Instructional Purposes," pp. 39–42.
2. Bell, Kate, "Practical Applications of Mathematics in Forestry," pp. 42–43.
3. Bordelon, W. J. "Will the Use of Applications Effect a More Functional Program of General Mathematics?" pp. 43–44.
4. Garrett, H. L., "Concrete Applications as Aids in Teaching Algebra and Geometry," pp. 44–45.
5. Kaltenborn, H. S., "Applications of Arithmetic and Geometric Progressions," pp. 45–46.
6. Mossman, Thirza, "Discrimination and Economy in the Use of Applications," pp. 46–47.
7. Rickey, F. A., "Curiosity—Satisfying Applications of Mathematics," pp. 47–48.
8. Weimer, M. Bird, "Applications of Some Topics in Mathematics," pp. 48–49.
9. Beito, E. A., and Read, C. B., "The Seventh December Meeting of the National Council of Teachers of Mathematics," pp. 49–50.

National Mathematics Magazine

November 1941, Vol. 16, No. 2

1. Sanders, S. T., "Mathematics and Nature," p. 58.
2. Dorwart, H. L., "Concerning a Certain Web of Conic," pp. 59–61.
3. Baten, William Dowell, "Comments Concerning a Note on Observed Geometric Series by A. B. Soble in the April 1940 Issue," pp. 62–63.
4. Finkel, Benjamin F., "A History of American Mathematical Journals" (Continued), pp. 64–78.
5. Bateman, H., "The Resistance of Ships," pp. 79–88.
6. Georges, J. S., "Educational Interests of Teachers of College Mathematics," pp. 89–90.
7. Georges, J. S., "Types of Learning Products of Evaluation of Instruction," pp. 90–101.

School Science and Mathematics

November, 1941, Vol. 41, No. 8

1. Dubisch, Roy, "A Mathematical Approach to Aesthetics, pp. 718–723.
2. Dohner, H. Ivol, "On a Problem of Steinhaus," pp. 765–767.
3. Stewart, Lyle F., "Teacher-Pupil Appraisals of 150 Science and Mathematics Films," pp. 769–774.
4. Hart, William L., "Mathematical Education for Defense," pp. 779–787.
5. Brown, Ernest N., "Integral Right Triangles," pp. 799–800.

New Books Received

A SURVEY OF MODERN ALGEBRA, by Garrett Birkoff and Saunders MacLane. Pp. xi+449. September 1941. The Macmillan Company. Price $3.75.

MATHEMATICS (Industrial Series), by John W. Breneman. Pp. xii+210. 1941. The McGraw-Hill Book Company. Price $1.75.

MECHANICS (Industrial Series), by John W. Breneman. Pp. x+141. 1941. The McGraw-Hill Book Company. Price $1.50.

STRENGTH OF MATERIALS (Industrial Series), by John W. Breneman. Pp. x+145. 1941. The McGraw-Hill Book Company. Price $1.50.

HANDBOOK OF MATHEMATICAL TABLES AND FORMULAS, by Burington, Richard Stevens Pp. 275. Handbook Publishers, Inc., Sandusky, Ohio. Second Edition, 1940.

THE TEACHING OF SECONDARY MATHEMATICS, by Charles H. Butler and Lynwood Wren. October 1941. McGraw-Hill Book Company. Pp. xii+514. Price $3.00.

BRIEF TRIGONOMETRY, by Edward A. Cameron. Pp. viii+138. 1941. Cornwall Press, Cornwall, New York. Price $1.25.

WHAT IS MATHEMATICS?, by Richard Courant and Herbert Robbins. 1941. Pp. xix+521. Oxford University Press. Price $3.75.

COLLEGE GEOMETRY, by Paul H. Daus. Pp. xv+200. 1941. Prentice-Hall, Inc. Price $2.50.

GALOIS LECTURES (The Scripta Mathematics Library Number Five), by Jessee Douglas, Philip Franklin, Cassius Jackson Keyser, Leopold Infeld. Pp. 124. 1941. Printed at the Morrill Press, Fulton, New York. Price $1.25.

MATHEMATICAL TABLES, by Herbert Bristol Dwight. 1941. The McGraw-Hill Book Company, Inc. Pp. vii+231. Price $2.50.

ELEMENTARY FUNCTIONS AND APPLICATION, by Arthur Sullivan Gale and Charles William Watkeys. Pp. xxi+409. 1941. Henry Holt and Company. Price $2.25.

ELEMENTS OF THE DIFFERENTIAL AND INTEGRAL CALCULUS by William Anthony Granville, Percey F. Smith, and William Raymond Longley. Pp. xii+556. 1941. Ginn and Company. Price $3.60.

MATHEMATICS TEACHERS' VIEWS ON CERTAIN ISSUES IN THE TEACHING OF MATHEMATICS, by Homer Howard. Pp. vii+134. May 1941. Bureau of Publication, Teachers College, Columbia University.

EARLY MILITARY BOOKS IN THE UNIVERSITY OF MICHIGAN LIBRARIES, by Thomas M. Spaulding and Louis C. Karpinski. 1941. Pp. xvi+371. The University of Michigan Press. Price $2.00.

SENIOR PRACTICAL MATHEMATICS, by N. J. Lennes. Pp. 584. The Macmillan Company. 1941. Price $1.80.

MATHEMATICS FOR ELECTRICIANS, by Martin H. Kuehn. Pp. xi+254. 1941. The McGraw-Hill Book Company, Inc. Price $1.75.

INTERMEDIATE ALGEBRA, by Charles H. Mergendahl and Thomas G. Walters. Pp. ix +434. 1941. D. Appleton-Century Company Inc. Price $1.48.

PRACTICAL MATHEMATICS, PART IV, by Claude Irwin Palmer and Samuel Fletcher Bibb. Pp. xii+209. 1941. The McGraw-Hill Book Company, Inc. Price $1.25.

CALCULUS, PART 1, by H. B. Phillips. Pp. v +229. 1940. Lew A. Cummings Company.

CALCULUS PART II, by H. B. Phillips. 1941. Pp. vi+457. Lew A. Cummings, Company, Cambridge, Massachusetts.

ELEMENTS OF AERONAUTICS, by Francis Pope and Arthur S. Otis. Pp. x+660. World Book Company. 1941.

ELEMENTARY LOGIC, by Willard Van Orman Quine. Pp. vi+170. 1941. Ginn and Company. Price $2.25.

FUNDAMENTALS OF MATHEMATICS, by M. Richardson. Pp. xviii+525. 1941. Macmillan Company. Price $3.25.

SELF-HELP MATHEMATICS WORK-BOOK Two, by G. M. Ruch, F. B. Knight, and J. W. Studebaker. 1941. Pp. 174. Scott, Foresman and Company.

BUSINESS ARITHMETIC, by Henry Smithline and Clyde O. Thompson. Pp. xi+496. 1941. Prentice-Hall, Inc. Price $1.60.

HIGHER MATHEMATICS FOR ENGINEERS AND PHYSICISTS, by Ivan S. Sokolnikoff and Elizabeth S. Sokolnikoff. Pp. xi+587. McGraw-Hill Book Company. Price $4.50. October 1941.

THE ANALYTICAL FOUNDATIONS OF CELESTIAL MECHANICS, by Aurel Wintner. Pp. xii+448. Princeton University Press. Price $6.00. 1941.

INTRODUCTION TO NON-EUCLIDEAN GEOMETRY, by H. E. Wolfe. Pp. ix+117. 1941. Printed by Edward Brothers, Inc.

TOOLS A MATHEMATICAL SKETCH AND MODEL BOOK, by Dr. Robert C. Yates. 1941. Louisiana State University. Pp. 1941. Price, $1.50 + Postage.